

Your First Math Book: BASIC MATH 101

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Made with AI Assistance

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*This book is dedicated to anyone who believes math is hard.
You've got this!*

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Part I

The Building Blocks – Numbers and Counting

Chapter 1

What Are Numbers?

1.1 Introduction to Numbers: Quantity, Symbols, and More!

Welcome to the exciting world of numbers! Think about it: numbers are everywhere! When you count your fingers, tell someone how old you are, or ask for two cookies, you're using numbers. But what exactly are they?

At their core, numbers represent **quantity** – how many of something there are. They help us keep track of things. We also use special **symbols** (like 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to write down these quantities.

Real-World Example:

You have 5 apples. The number '5' tells us exactly how many apples you possess.

1.2 Counting from 0 to 100 (and Beyond)

Counting is one of the first and most important math skills you'll learn. It's the process of assigning numbers to objects in a group to determine their quantity.

Let's start with the basics:

- 0: Represents nothing. If you have 0 cookies, you have no cookies.
- 1: One cookie.
- 2: Two cookies.
- ...and so on!

Practice counting forwards from 0 to 100. Then try counting backward from 100 to 0! This helps build a strong number sense.

Why is this important?

Counting isn't just for kids! Even in advanced math, counting principles are used in areas like probability and statistics to count possibilities and outcomes.

1.3 Number Line Basics

The number line is a super helpful tool in math. It's a straight line where numbers are placed at equal intervals. It helps us visualize numbers and understand their order.

(Imagine a straight line with numbers -3 to 3 marked at equal intervals, with 0 in the middle.)

Step 1: **Draw a straight line:** Use a ruler for neatness.

Step 2: **Mark a point for 0:** This is often in the middle.

Step 3: **Mark equal intervals to the right:** Label them 1, 2, 3, 4, ...

Step 4: **Mark equal intervals to the left:** Label them -1, -2, -3, ... (we'll cover these later!)

1.4 Place Value: Ones, Tens, Hundreds

Numbers can be big! How do we know the difference between 5, 50, and 500? It's all about **place value**. The position of a digit in a number tells us its value.

Let's look at the number 345:

- The '5' is in the **Ones place**. It means 5 units of 1.
- The '4' is in the **Tens place**. It means 4 groups of 10, which is 40.
- The '3' is in the **Hundreds place**. It means 3 groups of 100, which is 300.

So, 345 is really $300 + 40 + 5$.

Real-World Example:

If you have \$237, you have two hundreds, three tens, and seven ones. Knowing place value helps you understand the true worth of large numbers.

1.5 Comparing Numbers: Greater Than, Less Than, Equal To

Sometimes we need to know if one number is bigger, smaller, or the same as another. We use special symbols for this:

- $>$ (Greater than): The open side always points to the larger number. Example: $5 > 3$ (5 is greater than 3)
- $<$ (Less than): The closed side always points to the smaller number. Example: $2 < 7$ (2 is less than 7)
- $=$ (Equal to): The numbers on both sides are the same. Example: $4 = 4$ (4 is equal to 4)

To compare numbers, especially larger ones, start by looking at the digit in the largest place value.

Step 1: **Count digits:** If numbers have different numbers of digits, the one with more digits is usually larger (e.g., $100 > 99$).

Step 2: **Compare from left to right:** If numbers have the same number of digits, start comparing the leftmost digit.

Step 3: ***Move to the next digit:*** If the digits are the same, move to the next digit to the right until you find a difference.

Example: Compare 567 and 549.

- Hundreds place: Both have '5'.
- Tens place: 6 (in 567) is greater than 4 (in 549).
- So, $567 > 549$.

1.6 Odd and Even Numbers

Numbers can be sorted into two basic categories:

- **Even numbers:** Can be divided into two equal groups with nothing left over. They always end in 0, 2, 4, 6, or 8.
- **Odd numbers:** Cannot be divided into two equal groups without a remainder. They always end in 1, 3, 5, 7, or 9.

Real-World Example:

If you have 6 cookies (an even number), you can give 3 to yourself and 3 to a friend, with none left over. If you have 5 cookies (an odd number), you can't share them equally without cutting one in half!

Practice Problems

1. What is the value of the digit '7' in the number 472?
2. Count backwards from 25 to 15.
3. Draw a number line from 0 to 10 and mark the number 6.
4. Use $>$, $<$, or $=$ to compare the following numbers:
 - $38 \underline{\hspace{1cm}} 83$
 - $101 \underline{\hspace{1cm}} 99$
 - $50 + 5 \underline{\hspace{1cm}} 55$
5. Is 47 an odd or even number? How about 120?

Chapter 2

All About Addition (+) and Subtraction (-)

2.1 Concept of Adding (+): Putting Together

Addition is one of the most fundamental operations in mathematics. When you add, you are combining two or more groups of items to find the total sum. The symbol for addition is a plus sign (+).

Real-World Example:

If you have 3 red apples and 2 green apples, you can add them together to find out how many apples you have in total: 3 apples + 2 apples = 5 apples.

Why is this important?

Addition is commutative! This means you can add numbers in any order, and the sum will be the same. For example, $3 + 2 = 5$ and $2 + 3 = 5$.

2.2 Concept of Subtracting (-): Taking Away, Finding Difference

Subtraction is the opposite of addition. When you subtract, you are taking away a quantity from a larger quantity, or finding the difference between two quantities. The symbol for subtraction is a minus sign (-).

Real-World Example:

You have 7 cookies and you eat 3 of them. How many are left? 7 cookies - 3 cookies = 4 cookies.

Or, if you have 8 toys and your friend has 5 toys, what is the difference? 8 toys - 5 toys = 3 toys.

2.3 Addition Facts (0-10, 0-20)

Mastering your addition facts is like building a strong foundation for a house. Knowing these by heart will make all future math much easier.

Example: Addition Facts for 5 $5 + 0 = 5$ $5 + 1 = 6$ $5 + 2 = 7$ $5 + 3 = 8$ $5 + 4 = 9$ $5 + 5 = 10$...and so on!

2.4 Subtraction Facts (0-10, 0-20)

Just like addition facts, knowing your subtraction facts instantly is super helpful. Subtraction facts are directly related to addition facts. If you know $3 + 2 = 5$, then you also know $5 - 2 = 3$ and $5 - 3 = 2$.

Example: Subtraction Facts related to 10 $10 - 0 = 10$ $10 - 1 = 9$ $10 - 2 = 8$...and so on!

2.5 Adding and Subtracting Multi-Digit Numbers (with and without Regrouping/Borrowing)

When numbers get bigger, we use a special method called "column addition" or "column subtraction," which relies on our understanding of place value.

Adding Multi-Digit Numbers (No Regrouping)

Step 1: ****Line them up:**** Write the numbers one below the other, making sure the ones digits, tens digits, hundreds digits, etc., are perfectly aligned in columns.

Step 2: ****Add the ones:**** Start by adding the digits in the ones column. Write the sum below the line in the ones column.

Step 3: ****Add the tens:**** Move to the tens column and add those digits. Write the sum below the line in the tens column.

Step 4: ****Continue:**** Repeat for hundreds, thousands, and so on, until all columns are added.

Example: Add $23 + 45$

$$\begin{array}{r} 23 \\ + 45 \\ \hline 68 \end{array}$$

(Explanation: $3 + 5 = 8$ in the ones column. $2 + 4 = 6$ in the tens column.)

Adding Multi-Digit Numbers (With Regrouping / "Carrying Over")

Sometimes, when you add a column, the sum is 10 or more. This is when you "regroup" or "carry over."

Step 1: ****Line them up:**** Align the numbers by place value.

Step 2: ****Add the ones:**** Add the digits in the ones column.

Step 3: ****Regroup if sum is 10 or more:**** If the sum is 10 or greater, write the ones digit of the sum in the ones column and "carry over" the tens digit to the top of the tens column.

Step 4: ****Add the tens (including carry-over):**** Add the digits in the tens column, remembering to include any digit you carried over. Regroup if necessary.

2.5. ADDING AND SUBTRACTING MULTI-DIGIT NUMBERS (WITH AND WITHOUT REGROUPING)

Step 5: ****Continue:**** Repeat for hundreds, thousands, etc.

Example: Add $37 + 25$

$$\begin{array}{r} ^1 37 \text{ (Carry 1 to the tens place)} \\ + 25 \\ \hline 62 \end{array}$$

(Explanation: $7 + 5 = 12$. Write down '2', carry '1' to the tens place. Then $1 + 3 + 2 = 6$ in the tens column.)

Subtracting Multi-Digit Numbers (No Regrouping)

Step 1: ****Line them up:**** Align numbers by place value.

Step 2: ****Subtract the ones:**** Start with the ones column. Subtract the bottom digit from the top digit.

Step 3: ****Subtract the tens:**** Move to the tens column and subtract those digits.

Step 4: ****Continue:**** Repeat for hundreds, thousands, and so on.

Example: Subtract $86 - 32$

$$\begin{array}{r} 86 \\ - 32 \\ \hline 54 \end{array}$$

(Explanation: $6 - 2 = 4$ in the ones column. $8 - 3 = 5$ in the tens column.)

Subtracting Multi-Digit Numbers (With Regrouping / "Borrowing")

Sometimes the top digit in a column is smaller than the bottom digit. This is when you need to "regroup" or "borrow" from the next place value to the left.

Step 1: ****Line them up:**** Align numbers by place value.

Step 2: ****Subtract the ones:**** If the top digit in the ones column is smaller than the bottom digit, borrow 1 ten (which is 10 ones) from the tens place. The tens digit decreases by 1, and the ones digit increases by 10.

Step 3: ****Subtract the tens:**** Now subtract the digits in the ones column. Then move to the tens column. If borrowing was done, remember the changed tens digit. If the top tens digit is now smaller than the bottom, borrow from the hundreds place.

Step 4: ****Continue:**** Repeat for hundreds, thousands, etc.

Example: Subtract $52 - 27$

$$\begin{array}{r} \cancel{5}^4 \cancel{2}^{12} \text{ (Borrowed 1 ten from 5, making it 4, and added 10 to 2, making it 12)} \\ - 27 \\ \hline 25 \end{array}$$

(Explanation: You can't do $2 - 7$. Borrow 1 ten from the 5 (making it 4). The 2 becomes 12. Now, $12 - 7 = 5$. In the tens column, $4 - 2 = 2$.)

2.6 Word Problems: Simple Addition and Subtraction Scenarios.

Math is not just about numbers on a page; it's about solving real-world puzzles! Word problems help us practice translating everyday situations into mathematical equations.

Step 1: **Read Carefully:** Understand what the problem is asking.

Step 2: **Identify Keywords:** Look for words that tell you whether to add or subtract (e.g., "total," "sum," "in all" usually mean addition; "left," "difference," "how many more/fewer" usually mean subtraction).

Step 3: **Set up the Equation:** Write down the math problem.

Step 4: **Solve:** Do the calculation.

Step 5: **Check:** Does your answer make sense in the context of the problem?

Real-World Example:

Sarah has 12 stickers. Her friend gives her 8 more stickers. How many stickers does Sarah have in total?

- Keywords: "more," "in total" suggest addition.
- Equation: $12 + 8 = ?$
- Solution: $12 + 8 = 20$
- Answer: Sarah has 20 stickers in total.

Real-World Example:

A baker made 45 cupcakes. He sold 23 of them. How many cupcakes are left?

- Keywords: "sold," "left" suggest subtraction.
- Equation: $45 - 23 = ?$
- Solution: $45 - 23 = 22$
- Answer: The baker has 22 cupcakes left.

Practice Problems

1. Calculate: $15 + 7$
2. Calculate: $28 - 9$
3. Add: $146 + 32$
4. Subtract: $58 - 25$

2.6. WORD PROBLEMS: SIMPLE ADDITION AND SUBTRACTION SCENARIOS.9

5. Add: $39 + 48$
6. Subtract: $73 - 18$
7. Maria had \$25. She earned \$15 more doing chores. How much money does Maria have now?
8. There are 30 birds on a tree. 12 birds fly away. How many birds are left on the tree?

Part II

Mastering the Core Operations – Multiplication and Division

Chapter 3

Introduction to Multiplication (*)

3.1 Concept of Multiplication (*): Repeated Addition, Equal Groups

Multiplication is a faster way to do repeated addition when you have equal groups. It's like adding the same number many times. The symbol for multiplication is usually a cross ($\times$) or an asterisk (*), or sometimes a dot ($\cdot$).

Real-World Example:

If you have 3 boxes, and each box has 4 chocolates inside, how many chocolates do you have in total? You could do $4 + 4 + 4 = 12$. Or, using multiplication: $3 \times 4 = 12$. This means "3 groups of 4" or "4 taken 3 times."

3.2 Multiplication Facts (Times Tables up to 12x12)

Learning your multiplication facts (times tables) is crucial! It will speed up your calculations tremendously. Practice makes perfect!

Why is this important?

Multiplication is also commutative! Just like addition, the order of the numbers doesn't change the product. $3 \times 4 = 12$ and $4 \times 3 = 12$.

3.3 Multiplying by 10, 100, 1000

Multiplying by powers of 10 is very straightforward!

- To multiply by 10, add one zero to the end of the number. (e.g., $7 \times 10 = 70$)
- To multiply by 100, add two zeros to the end of the number. (e.g., $7 \times 100 = 700$)
- To multiply by 1000, add three zeros to the end of the number. (e.g., $7 \times 1000 = 7000$)

3.4 Multiplying Multi-Digit Numbers by Single-Digit Numbers

When multiplying larger numbers, we use a process similar to addition and subtraction, using place value.

Step 1: ****Line them up:**** Write the multi-digit number on top, and the single-digit number below it, aligning the ones digits.

Step 2: ****Multiply the ones:**** Multiply the single-digit number by the ones digit of the top number. If the product is 10 or more, write down the ones digit of the product and carry over the tens digit.

Step 3: ****Multiply the tens:**** Multiply the single-digit number by the tens digit of the top number. Add any carried-over digit. Regroup if necessary.

Step 4: ****Continue:**** Repeat for hundreds, thousands, etc., until all digits in the top number have been multiplied.

Example: Multiply 43×2

$$\begin{array}{r} 43 \\ \times 2 \\ \hline 86 \end{array}$$

(Explanation: $2 \times 3 = 6$ (ones). $2 \times 4 = 8$ (tens).)

Example with Regrouping: Multiply 27×3

$$\begin{array}{r} 27 \\ \times 3 \\ \hline 81 \end{array}$$

(Explanation: $3 \times 7 = 21$. Write down '1', carry '2'. Then $3 \times 2 = 6$. Add the carried '2': $6 + 2 = 8$. Result is 81.)

3.5 Multiplying Two-Digit Numbers by Two-Digit Numbers (Area Model, Standard Algorithm)

This is a bit more involved but follows logical steps.

Area Model (Visualizing Multiplication)

The area model breaks down the multiplication into smaller, easier problems based on place value.

Example: Multiply 12×13

- Break down 12 into $10 + 2$.
- Break down 13 into $10 + 3$.

3.5. MULTIPLYING TWO-DIGIT NUMBERS BY TWO-DIGIT NUMBERS (AREA MODEL, STANDARD ALGORITHM)

- Draw a box and divide it into four smaller boxes:
- Multiply for each box:
 - $10 \times 10 = 100$
 - $10 \times 3 = 30$
 - $2 \times 10 = 20$
 - $2 \times 3 = 6$
- Add all the products: $100 + 30 + 20 + 6 = 156$.

Standard Algorithm (Column Multiplication)

This is the most common method taught for multiplying multi-digit numbers.

Step 1: ****Line them up:**** Align the numbers by their rightmost digits.

Step 2: ****Multiply by the ones digit:**** Multiply the bottom number's ones digit by each digit of the top number, starting from the right. Write down products and carry over if necessary. This gives you the first "partial product."

Step 3: ****Place a zero placeholder:**** Before multiplying by the tens digit, place a zero in the ones column of the next row (because you're now multiplying by tens, not ones).

Step 4: ****Multiply by the tens digit:**** Multiply the bottom number's tens digit by each digit of the top number, starting from the right. Write down products and carry over if necessary. This gives you the second "partial product."

Step 5: ****Add the partial products:**** Add the two partial products together to get your final answer.

Example: Multiply 24×32

$$\begin{array}{r} 24 \\ \times 32 \\ \hline 48 \quad (24 \times 2) \\ +720 \quad (24 \times 30; \text{note the zero placeholder}) \\ \hline 768 \end{array}$$

Practice Problems

1. Use repeated addition to find the product: 5×3
2. Complete the multiplication fact: $7 \times 8 = ?$
3. Multiply: 25×100
4. Calculate: 53×4
5. Calculate: 13×21 (Try both the area model and standard algorithm!)
6. A classroom has 6 tables. Each table has 4 chairs. How many chairs are there in total?

Chapter 4

Understanding Division (/)

4.1 Concept of Division (/): Sharing Equally, Repeated Subtraction

Division is the process of splitting a number into equal parts or groups. It's the inverse (opposite) of multiplication. The symbol for division can be $\div$, $/$, or a fraction bar.

Real-World Example:

You have 10 cookies, and you want to share them equally among 5 friends. How many cookies does each friend get? $10 \div 5 = 2$. Each friend gets 2 cookies. This is like repeatedly subtracting 5 from 10 until nothing is left ($10 - 5 = 5$, $5 - 5 = 0$). You did it 2 times!

4.2 Division Facts (Related to Multiplication Facts)

Just like subtraction relates to addition, division facts are directly related to multiplication facts. If you know $3 \times 4 = 12$, then you know $12 \div 4 = 3$ and $12 \div 3 = 4$.

4.3 Dividing with Remainders

Sometimes, when you divide, the numbers don't split perfectly into equal groups. The amount left over is called the **remainder**.

Real-World Example:

You have 11 apples and want to share them equally among 3 friends. $11 \div 3 = ?$ You can give each friend 3 apples ($3 \times 3 = 9$). You have $11 - 9 = 2$ apples left over. So, $11 \div 3 = 3$ with a remainder of 2. We can write this as $3R2$.

4.4 Long Division (Single-Digit Divisor, Then Two-Digit Divisor)

Long division is a method used for dividing larger numbers. It breaks down the division into a series of smaller, manageable steps.

Long Division with a Single-Digit Divisor

Let's divide $78 \div 3$.

Step 1: **Divide:** How many times does 3 go into 7? (The first digit of the dividend). It goes 2 times. Write '2' above the '7'.

Step 2: **Multiply:** Multiply the quotient digit (2) by the divisor (3): $2 \times 3 = 6$. Write '6' below the '7'.

Step 3: **Subtract:** Subtract 6 from 7: $7 - 6 = 1$.

Step 4: **Bring Down:** Bring down the next digit of the dividend (8) next to the 1, making it 18.

Step 5: **Repeat:** Now, repeat the process with 18.

- Divide: How many times does 3 go into 18? It goes 6 times. Write '6' next to the '2' in the quotient.
- Multiply: $6 \times 3 = 18$. Write '18' below the '18'.
- Subtract: $18 - 18 = 0$.
- Bring Down: No more digits to bring down.

So, $78 \div 3 = 26$.

Visual representation of long division (standard notation):

$$\begin{array}{r}
 26 \\
 \text{----} \\
 3 \overline{) 78} \\
 \underline{-6} \quad (2 * 3) \\
 \text{---} \\
 18 \\
 \underline{-18} \quad (6 * 3) \\
 \text{---} \\
 0
 \end{array}$$

Long Division with a Two-Digit Divisor

The process is the same, but estimating how many times the divisor goes into the partial dividend can be trickier.

Example: Divide $364 \div 13$.

Step 1: **Divide:** How many times does 13 go into 36? (Since 13 won't go into 3, look at 36). Estimate: $13 \times 1 = 13$, $13 \times 2 = 26$, $13 \times 3 = 39$. So, 2 times. Write '2' above the '6'.

Step 2: **Multiply:** $2 \times 13 = 26$. Write '26' below the '36'.

Step 3: **Subtract:** $36 - 26 = 10$.

Step 4: **Bring Down:** Bring down the next digit (4), making it 104.

Step 5: **Repeat:** How many times does 13 go into 104? Estimate: $13 \times 5 = 65$, $13 \times 8 = 104$. So, 8 times. Write '8' next to the '2'.

Step 6: ****Multiply:**** $8 \times 13 = 104$. Write '104' below the '104'.

Step 7: ****Subtract:**** $104 - 104 = 0$.

So, $364 \div 13 = 28$.

$$\begin{array}{r}
 28 \\
 \text{----} \\
 13 \overline{) 364} \\
 \underline{-26} \quad (2 * 13) \\
 104 \\
 \underline{-104} \quad (8 * 13) \\
 0
 \end{array}$$

4.5 Word Problems: Simple Multiplication and Division Scenarios.

Practice translating real-life situations into division or multiplication problems. Look for keywords: "each," "per," "share equally," "split" often mean division.

Practice Problems

1. Complete the division fact: $48 \div 6 = ?$
2. Divide: $25 \div 4$ (include remainder)
3. Calculate using long division: $96 \div 8$
4. Calculate using long division: $435 \div 15$
5. A baker made 72 cookies and wants to put them into bags, with 9 cookies per bag. How many bags does he need?
6. A car travels 60 miles per hour. How far does it travel in 3 hours?

Chapter 5

Order of Operations (PEMDAS/BODMAS)

5.1 Why We Need an Order

What if you see a math problem like $2 + 3 \times 4$? If you add first ($2 + 3 = 5$, then $5 \times 4 = 20$), you get 20. But if you multiply first ($3 \times 4 = 12$, then $2 + 12 = 14$), you get 14. Which one is correct? To avoid confusion and ensure everyone gets the same answer, mathematicians agreed on a specific order. This is called the **Order of Operations**.

5.2 PEMDAS / BODMAS

There are two common acronyms to remember the order:

- **PEMDAS:**
 - **P**arentheses (or Brackets)
 - **E**xponents (or Orders)
 - **M**ultiplication and **D**ivision (from left to right)
 - **A**ddition and **S**ubtraction (from left to right)
- **BODMAS:**
 - **B**rackets
 - **O**rders (powers/roots)
 - **D**ivision and **M**ultiplication (from left to right)
 - **A**ddition and **S**ubtraction (from left to right)

They mean the same thing! Multiplication/Division and Addition/Subtraction are done from left to right within their own "tier."

5.3 Parentheses/Brackets

Always do calculations inside parentheses () or brackets [] first. They act like a "do me first!" sign. Example: $(2 + 3) \times 4 = 5 \times 4 = 20$. (The answer from our earlier example!)

5.4 Exponents (Basic Introduction)

Exponents are a shorthand for repeated multiplication. For example, 2^3 means $2 \times 2 \times 2$. We'll cover these more in a later chapter, but for now, know that they come after parentheses.

5.5 Multiplication/Division (Left to Right)

After parentheses and exponents, perform all multiplication and division operations as you encounter them from left to right in the expression. They have equal priority. Example: $10 \div 2 \times 5$ If you did $\times$ first: $10 \div 10 = 1$ (Incorrect!) Correct: $10 \div 2 = 5$, then $5 \times 5 = 25$.

5.6 Addition/Subtraction (Left to Right)

Finally, perform all addition and subtraction operations as you encounter them from left to right. They also have equal priority. Example: $8 - 3 + 2$ If you did $+$ first: $8 - 5 = 3$ (Incorrect!) Correct: $8 - 3 = 5$, then $5 + 2 = 7$.

5.7 Solving Multi-Step Problems Using the Correct Order.

Let's put it all together!

Example: Solve $18 \div (3 + 3) - 2 \times 4$

Step 1: ****Parentheses first:**** $3 + 3 = 6$. So the problem becomes: $18 \div 6 - 2 \times 4$

Step 2: ****Multiplication/Division (left to right):****

- $18 \div 6 = 3$
- $2 \times 4 = 8$

So the problem becomes: $3 - 8$

Step 3: ****Addition/Subtraction (left to right):****

- $3 - 8 = -5$ (Introduction to negative numbers!)

Final Answer: -5 .

Practice Problems

1. Calculate: $5 + 2 \times 3$
2. Calculate: $(10 - 2) \div 4$
3. Calculate: $20 - 6 \div 2 + 5$
4. Calculate: $4 \times (3 + 1) - 10 \div 5$

5. Calculate: $3^2 + (8 - 4) \times 2$

Part III

Expanding the Number System – Fractions, Decimals, and Integers

Chapter 6

Deep Dive into Fractions

Fractions are numbers that represent parts of a whole or parts of a collection. Imagine a pizza cut into slices – each slice is a fraction of the whole pizza!

6.1 What Are Fractions? (Parts of a Whole, Parts of a Set)

A fraction is a way to express a quantity that is not a whole number. It consists of two parts: a numerator and a denominator.

Real-World Example:

If you cut a cake into 8 equal slices and eat 3 of them, you have eaten $\frac{3}{8}$ of the cake.

6.2 Numerator and Denominator

In a fraction $\frac{a}{b}$:

- The top number is the **numerator**. It tells you how many parts you have.
- The bottom number is the **denominator**. It tells you how many equal parts the whole is divided into.

Example: In $\frac{3}{8}$, the numerator is 3 (you have 3 slices), and the denominator is 8 (the cake was cut into 8 slices).

6.3 Types of Fractions (Proper, Improper, Mixed Numbers)

- **Proper Fraction:** The numerator is smaller than the denominator. It represents a value less than 1. Example: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{10}$
- **Improper Fraction:** The numerator is equal to or larger than the denominator. It represents a value equal to or greater than 1. Example: $\frac{5}{3}$, $\frac{7}{7}$, $\frac{11}{4}$
- **Mixed Number:** A combination of a whole number and a proper fraction. Example: $1\frac{2}{3}$, $3\frac{1}{2}$

- Step 1:** ****Converting Improper to Mixed:**** Divide the numerator by the denominator. The quotient is the whole number, the remainder is the new numerator, and the denominator stays the same. Example: $\frac{11}{4}$ $11 \div 4 = 2$ with a remainder of 3. So, $\frac{11}{4} = 2\frac{3}{4}$.
- Step 2:** ****Converting Mixed to Improper:**** Multiply the whole number by the denominator, add the numerator, and put the result over the original denominator. Example: $2\frac{3}{4}$ $(2 \times 4) + 3 = 8 + 3 = 11$. So, $2\frac{3}{4} = \frac{11}{4}$.

6.4 Equivalent Fractions and Simplifying Fractions

Equivalent fractions are different fractions that represent the same value. Example: $\frac{1}{2}$ is equivalent to $\frac{2}{4}$ and $\frac{3}{6}$.

To find equivalent fractions, multiply (or divide) both the numerator and the denominator by the same non-zero number. Example: $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

Simplifying fractions (or reducing to lowest terms) means finding an equivalent fraction where the numerator and denominator have no common factors other than 1.

Step 1: ****Find the Greatest Common Factor (GCF):**** This is the largest number that divides evenly into both the numerator and denominator.

Step 2: ****Divide:**** Divide both the numerator and denominator by their GCF.

Example: Simplify $\frac{6}{8}$. Factors of 6: 1, 2, 3, 6 Factors of 8: 1, 2, 4, 8 The GCF is 2. $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$.

6.5 Comparing and Ordering Fractions

To compare fractions, you need a common ground.

Step 1: ****Find a Common Denominator (LCD):**** Find the Least Common Multiple (LCM) of the denominators. This will be your new common denominator.

Step 2: ****Convert Fractions:**** Convert each fraction to an equivalent fraction with the common denominator.

Step 3: ****Compare Numerators:**** The fraction with the larger numerator is the greater fraction.

Example: Compare $\frac{2}{3}$ and $\frac{3}{4}$. LCM of 3 and 4 is 12. $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ Since $9 > 8$, $\frac{9}{12} > \frac{8}{12}$, so $\frac{3}{4} > \frac{2}{3}$.

6.6 Adding and Subtracting Fractions (Like and Unlike Denominators)

Like Denominators

If fractions have the same denominator, adding or subtracting is easy! Just add or subtract the numerators and keep the denominator the same. Example: $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$

Example: $\frac{4}{7} - \frac{1}{7} = \frac{4-1}{7} = \frac{3}{7}$

Unlike Denominators

If fractions have different denominators, you must find a common denominator first, then add or subtract.

Step 1: **Find LCD:** Find the Least Common Denominator (same as LCM).

Step 2: **Convert:** Convert each fraction to an equivalent fraction with the LCD.

Step 3: **Add/Subtract:** Add or subtract the new numerators. Keep the LCD.

Step 4: **Simplify:** Simplify the result if possible.

Example: $\frac{1}{2} + \frac{1}{3}$ LCD of 2 and 3 is 6. $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ $\frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$

6.7 Multiplying and Dividing Fractions

Multiplying Fractions

Multiplying fractions is straightforward: multiply the numerators together and multiply the denominators together. Simplify the result. Example: $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$

Dividing Fractions

To divide fractions, use the "Keep, Change, Flip" method:

- ****Keep**** the first fraction as it is.
- ****Change**** the division sign to a multiplication sign.
- ****Flip**** (invert) the second fraction (swap its numerator and denominator).
- Then, ****Multiply**** the fractions.

Example: $\frac{1}{2} \div \frac{3}{4}$

$$\begin{aligned} \frac{1}{2} \div \frac{3}{4} &= \frac{1}{2} \times \frac{4}{3} \quad (\text{Keep } \frac{1}{2}, \text{ Change } \div \text{ to } \times, \text{ Flip } \frac{3}{4} \text{ to } \frac{4}{3}) \\ &= \frac{1 \times 4}{2 \times 3} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \quad (\text{Simplify}) \end{aligned}$$

Practice Problems

1. What fraction of the shapes are stars? (Imagine 3 stars, 2 circles)
2. Identify the numerator and denominator in $\frac{5}{9}$.
3. Convert $\frac{17}{5}$ to a mixed number.
4. Convert $3\frac{1}{4}$ to an improper fraction.

5. Simplify the fraction $\frac{10}{15}$.
6. Are $\frac{2}{5}$ and $\frac{4}{10}$ equivalent?
7. Compare $\frac{1}{4}$ and $\frac{2}{3}$ using $<$ or $>$.
8. Add: $\frac{3}{8} + \frac{2}{8}$
9. Subtract: $\frac{7}{10} - \frac{1}{5}$
10. Multiply: $\frac{3}{4} \times \frac{1}{2}$
11. Divide: $\frac{5}{6} \div \frac{2}{3}$

Chapter 7

Working with Decimals

Decimals are another way to represent parts of a whole, especially useful when dealing with money, measurements, and percentages. They are based on powers of ten.

7.1 What Are Decimals? (Parts of 10, 100, etc.)

A decimal number uses a decimal point (.) to separate the whole number part from the fractional part. The digits to the right of the decimal point represent fractions with denominators of 10, 100, 1000, and so on.

Real-World Example:

Money uses decimals! \$3.75 means 3 whole dollars and 75 hundredths of a dollar (or 75 cents).

7.2 Place Value in Decimals

Just like whole numbers, decimals have place value. For the number 123.456:

- 1: Hundreds place
- 2: Tens place
- 3: Ones place
- .: Decimal point
- 4: **Tenths** place ($\frac{1}{10}$)
- 5: **Hundredths** place ($\frac{1}{100}$)
- 6: **Thousandths** place ($\frac{1}{1000}$)

Notice the "th" at the end for decimal places!

7.3 Converting Between Fractions and Decimals

Fraction to Decimal

Step 1: ****Divide:**** Divide the numerator by the denominator.

Example: Convert $\frac{3}{4}$ to a decimal. $3 \div 4 = 0.75$

Decimal to Fraction

Step 1: ****Write the decimal as a fraction over a power of 10:**** The denominator will be 10, 100, 1000, etc., depending on the last decimal place.

Step 2: ****Simplify:**** Reduce the fraction to its simplest form.

Example: Convert 0.6 to a fraction. 0.6 is "six tenths", so $\frac{6}{10}$. Simplify: $\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$.

7.4 Comparing and Ordering Decimals

Step 1: ****Line up decimal points:**** Write the numbers vertically, aligning the decimal points.

Step 2: ****Add zeros:**** Add zeros to the end of the shorter decimal numbers so they all have the same number of decimal places. (This doesn't change their value).

Step 3: ****Compare:**** Compare the numbers as if they were whole numbers, starting from the leftmost digit.

Example: Compare 0.5 and 0.45. Line up: 0.50 (added a zero) 0.45 Compare: 50 is greater than 45, so $0.5 > 0.45$.

7.5 Adding and Subtracting Decimals

This is similar to adding and subtracting whole numbers, but with one crucial rule:

Step 1: ****Line up decimal points:**** Place the numbers one below the other so that the decimal points are perfectly aligned.

Step 2: ****Add zeros:**** Add zeros to the end of numbers so all columns have a digit.

Step 3: ****Add/Subtract:**** Perform the operation as you would with whole numbers.

Step 4: ****Place decimal point:**** Bring the decimal point straight down into the answer.

Example: Add $1.25 + 0.3$

$$\begin{array}{r} 1.25 \\ + 0.30 \text{ (added a zero)} \\ \hline 1.55 \end{array}$$

7.6 Multiplying and Dividing Decimals (Including by Powers of 10)

Multiplying Decimals

Step 1: ****Ignore decimal points:**** Multiply the numbers as if they were whole numbers.

Step 2: ****Count decimal places:**** Count the total number of decimal places in both numbers being multiplied.

Step 3: ****Place decimal point:**** In your product, start from the right and count left the total number of decimal places you found in Step 2. Place the decimal point there.

Example: 2.5×0.3

- Multiply $25 \times 3 = 75$.
- 2.5 has 1 decimal place. 0.3 has 1 decimal place. Total: $1 + 1 = 2$ decimal places.
- Start from right in 75, move 2 places left: 0.75.

Multiplying/Dividing by Powers of 10 (10, 100, 1000...)

- ****Multiplying:**** Move the decimal point to the **right** for each zero in the power of 10. Example: $3.45 \times 10 = 34.5$ (move 1 place right) Example: $3.45 \times 100 = 345$ (move 2 places right)
- ****Dividing:**** Move the decimal point to the **left** for each zero in the power of 10. Example: $3.45 \div 10 = 0.345$ (move 1 place left) Example: $3.45 \div 100 = 0.0345$ (move 2 places left)

Dividing Decimals

Step 1: ****Make divisor a whole number:**** Move the decimal point in the divisor (the number you are dividing by) to the right until it is a whole number.

Step 2: ****Move dividend's decimal:**** Move the decimal point in the dividend (the number being divided) the SAME number of places to the right. Add zeros if needed.

Step 3: ****Divide:**** Perform long division as with whole numbers.

Step 4: ****Place decimal point:**** Bring the decimal point straight up into the quotient (your answer).

Example: $4.8 \div 0.6$

- Move decimal in 0.6 one place right to get 6.
- Move decimal in 4.8 one place right to get 48.
- Now divide $48 \div 6 = 8$.

7.7 Rounding Decimals

Rounding decimals is similar to rounding whole numbers.

Step 1: ****Identify the rounding place:**** This is the place value you're asked to round to (e.g., nearest tenth, nearest hundredth).

Step 2: ****Look at the digit to its right:****

- If this digit is 5 or greater (5, 6, 7, 8, 9), round up the digit in the rounding place.

- If this digit is less than 5 (0, 1, 2, 3, 4), keep the digit in the rounding place the same.

Step 3: ****Drop digits:**** Remove all digits to the right of the rounding place.

Example: Round 3.14159 to the nearest hundredth.

- Rounding place: hundredths (the '4').
- Digit to the right: '1'.
- Since '1' is less than 5, keep the '4' as it is.
- Drop digits after the '4'.

Result: 3.14

Practice Problems

1. What is the place value of the digit '2' in 5.128?
2. Convert $\frac{1}{4}$ to a decimal.
3. Convert 0.75 to a simplified fraction.
4. Order from least to greatest: 0.1, 0.09, 0.11
5. Add: $12.3 + 4.75$
6. Subtract: $5.0 - 2.8$
7. Multiply: 0.7×0.4
8. Divide: $25.5 \div 5$
9. Multiply: 6.2×100
10. Divide: $14.7 \div 10$
11. Round 7.863 to the nearest tenth.

Chapter 8

Exploring Integers (Positive and Negative Numbers)

So far, we've mostly worked with numbers that are zero or greater. But what about numbers below zero? These are called negative numbers, and together with positive numbers and zero, they make up the set of **integers**.

8.1 What Are Integers? (Number Line, Real-World Examples: Temperature, Debt)

Integers are all whole numbers (no fractions or decimals) and their opposites. Examples: ..., -3, -2, -1, 0, 1, 2, 3, ...

Why is this important?

Zero is an integer, but it is neither positive nor negative.

Real-World Example:

- **Temperature:** -5 degrees Celsius means 5 degrees below zero.
- **Debt:** If you owe someone \$10, you could say you have -\$10.
- **Elevation:** Below sea level (e.g., -100 feet).
- **Sports:** Losing yards in football (e.g., -8 yards).

Number Line for Integers

The number line extends in both directions. Positive numbers are to the right of zero, and negative numbers are to the left.

(Imagine a straight line extending from -3 to 3, with 0 in the middle.)

8.2 Comparing and Ordering Integers

On the number line, numbers to the right are always greater than numbers to the left. Example: $5 > 2$ (5 is to the right of 2) Example: $-1 > -3$ (-1 is to the right of -3, even though 3 looks "bigger" than 1) Example: $0 > -5$ (0 is to the right of -5)

8.3 Adding Integers

Adding integers has different rules depending on the signs.

- **Same Signs:** Add the absolute values and keep the original sign. Example: $3 + 5 = 8$ Example: $(-3) + (-5) = -8$
- **Different Signs:** Subtract the smaller absolute value from the larger absolute value. Keep the sign of the number with the larger absolute value. Example: $5 + (-3)$ Absolute values are 5 and 3. $5 - 3 = 2$. The '5' is positive and has a larger absolute value, so the answer is positive. $5 + (-3) = 2$. Example: $3 + (-5)$ Absolute values are 3 and 5. $5 - 3 = 2$. The '5' is negative and has a larger absolute value, so the answer is negative. $3 + (-5) = -2$.

8.4 Subtracting Integers

To subtract integers, **add the opposite**. Change the subtraction problem into an addition problem by changing the sign of the number being subtracted. Example: $5 - 3 = 5 + (-3) = 2$ Example: $3 - 5 = 3 + (-5) = -2$ Example: $5 - (-3) = 5 + 3 = 8$ (Subtracting a negative is like adding a positive!) Example: $-5 - (-3) = -5 + 3 = -2$

8.5 Multiplying and Dividing Integers (Rules for Signs)

The rules for signs are the same for multiplication and division.

- **Same Signs:** The result is ****positive****. Example: $3 \times 4 = 12$ Example: $(-3) \times (-4) = 12$ Example: $12 \div 3 = 4$ Example: $(-12) \div (-3) = 4$
- **Different Signs:** The result is ****negative****. Example: $(-3) \times 4 = -12$ Example: $3 \times (-4) = -12$ Example: $(-12) \div 3 = -4$ Example: $12 \div (-3) = -4$

8.6 Absolute Value

The **absolute value** of a number is its distance from zero on the number line, without regard to direction. It's always positive or zero. We use vertical bars to denote absolute value: $| \quad |$. Example: $|5| = 5$ (5 is 5 units away from zero) Example: $|-5| = 5$ (-5 is also 5 units away from zero) Example: $|0| = 0$

Practice Problems

1. Use $>$ or $<$ to compare: -7 ___ -2
2. What integer represents 10 degrees below zero?
3. Calculate: $10 + (-4)$
4. Calculate: $(-6) + (-8)$
5. Calculate: $7 - 12$

6. Calculate: $-3 - (-9)$
7. Calculate: $5 \times (-6)$
8. Calculate: $(-20) \div (-4)$
9. What is $|-15|$?
10. What is $|15| - |-5|$?

Part IV

Applied Arithmetic – Ratios, Proportions, Percentages, and Measurement

Chapter 9

Ratios, Rates, and Proportions

These concepts help us compare quantities and understand relationships between different values in the real world.

9.1 What Are Ratios? (Comparing Quantities)

A **ratio** is a comparison of two or more quantities. It tells you how much of one thing there is compared to another. Ratios can be written in a few ways:

- Using a colon: $a : b$
- Using the word "to": a to b
- As a fraction: $\frac{a}{b}$

It's important that the quantities are in the correct order.

Real-World Example:

In a basket, you have 3 red apples and 5 green apples. The ratio of red apples to green apples is $3 : 5$ (or 3 to 5, or $\frac{3}{5}$). The ratio of green apples to red apples is $5 : 3$ (or 5 to 3, or $\frac{5}{3}$). The ratio of red apples to total apples is $3 : 8$ (since $3 + 5 = 8$ total apples).

9.2 Equivalent Ratios

Just like equivalent fractions, **equivalent ratios** represent the same relationship between quantities. You can find equivalent ratios by multiplying or dividing both parts of the ratio by the same non-zero number. Example: The ratio $1 : 2$ is equivalent to $2 : 4$ (multiply by 2) and $5 : 10$ (multiply by 5).

9.3 What Are Rates? (Ratios with Different Units, e.g., Speed)

A **rate** is a special type of ratio that compares two quantities with *different* units. Example: miles per hour, dollars per pound, words per minute.

Real-World Example:

If you drive 120 miles in 2 hours, your rate of speed is $\frac{120 \text{ miles}}{2 \text{ hours}}$.

9.4 Unit Rates

A **unit rate** is a rate where the second quantity is 1 unit. This is often the most useful form for comparing different rates. To find a unit rate, divide the first quantity by the second quantity. Example: $\frac{120 \text{ miles}}{2 \text{ hours}} = 60 \text{ miles/hour}$. This is a unit rate because it tells you how many miles are driven in 1 hour.

Why is this important?

Unit rates are used all the time for "best buy" decisions in stores. If a 12-ounce box of cereal costs \$3 and a 16-ounce box costs \$4, which is the better deal? You'd compare their unit rates (cost per ounce).

9.5 What Are Proportions? (Equal Ratios)

A **proportion** is an equation that states that two ratios are equal. Example: $\frac{1}{2} = \frac{2}{4}$ is a proportion.

9.6 Solving Proportions (Cross-Multiplication)

Proportions are incredibly useful for finding unknown quantities. The most common method to solve a proportion is **cross-multiplication**. In a proportion $\frac{a}{b} = \frac{c}{d}$, cross-multiplication means $a \times d = b \times c$.

Step 1: **Set up the proportion:** Write the given ratios, with a variable (like x) for the unknown quantity.

Step 2: **Cross-multiply:** Multiply the numerator of the first ratio by the denominator of the second, and vice-versa.

Step 3: **Solve the equation:** You'll end up with a simple equation that you can solve for the variable.

Example: If 2 apples cost \$1, how much do 6 apples cost?

- Set up the proportion: $\frac{2 \text{ apples}}{\$1} = \frac{6 \text{ apples}}{\$x}$
- Cross-multiply: $2 \times x = 1 \times 6$
- Solve: $2x = 6$ Divide both sides by 2: $x = \frac{6}{2} = 3$.

So, 6 apples cost \$3.

9.7 Real-World Applications (Scaling Recipes, Map Reading)

Ratios and proportions are everywhere:

- **Recipes:** If a recipe calls for a ratio of 1 cup of flour to 2 cups of sugar, and you want to triple the recipe, you'd use proportions to find the new amounts.

- **Maps:** Map scales are ratios (e.g., 1 inch on the map = 100 miles in real life). You can use proportions to find real distances.
- **Photography:** Changing the size of an image while keeping its proportions.
- **Building/Design:** Scaling blueprints or models.

Practice Problems

1. Write the ratio of circles to squares if you have 4 circles and 6 squares (in three ways).
2. Find two equivalent ratios for 4 : 12.
3. If a car travels 200 miles in 4 hours, what is its rate in miles per hour?
4. A store sells 3 pencils for \$1.50. What is the unit price per pencil?
5. Solve the proportion for x : $\frac{x}{5} = \frac{9}{15}$
6. A recipe calls for 2 cups of sugar for every 3 cups of flour. If you use 6 cups of flour, how much sugar do you need?

Chapter 10

Understanding Percentages (%)

Percentages are a special kind of ratio that compares a number to 100. The word "percent" literally means "per one hundred" or "out of one hundred." The symbol for percent is %.

10.1 What Is a Percentage? (Parts Per 100)

A percentage tells you how many parts out of 100 you have. Example: 50% means 50 out of 100.

Real-World Example:

If you got 80% on a test, it means you got 80 questions right out of every 100 questions (or an equivalent proportion).

10.2 Converting Between Fractions, Decimals, and Percentages

These three forms are just different ways of representing the same value.

Percentage to Decimal

Step 1: **Divide by 100:** Remove the % sign and divide the number by 100 (move the decimal point 2 places to the left).

Example: $75\% = 75 \div 100 = 0.75$ Example: $5\% = 5 \div 100 = 0.05$

Decimal to Percentage

Step 1: **Multiply by 100:** Multiply the decimal by 100 (move the decimal point 2 places to the right) and add the % sign.

Example: $0.25 = 0.25 \times 100 = 25\%$ Example: $1.5 = 1.5 \times 100 = 150\%$

Percentage to Fraction

Step 1: **Write over 100:** Remove the % sign and write the number as a fraction with a denominator of 100.

Step 2: ****Simplify:**** Simplify the fraction to its lowest terms.

Example: $40\% = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$

Fraction to Percentage

Step 1: ****Convert to Decimal:**** Divide the numerator by the denominator.

Step 2: ****Convert Decimal to Percentage:**** Multiply the decimal by 100 and add the % sign.

Example: $\frac{3}{4} = 3 \div 4 = 0.75$. Then $0.75 \times 100 = 75\%$.

10.3 Finding a Percentage of a Number

To find a percentage of a number, convert the percentage to a decimal (or fraction) and multiply. Formula: Part = Percent (as decimal) $\times$ Whole

Real-World Example:

What is 20% of 80?

- Convert 20% to a decimal: 0.20.
- Multiply: $0.20 \times 80 = 16$.

So, 20% of 80 is 16.

10.4 Finding the Whole When Given a Part and a Percentage

Sometimes you know the part and the percentage, and you need to find the original whole amount. Formula: Whole = Part $\div$ Percent (as decimal)

Real-World Example:

15 is 30% of what number?

- Convert 30% to a decimal: 0.30.
- Divide: $15 \div 0.30 = 50$.

So, 15 is 30% of 50.

10.5 Percentage Increase and Decrease

These calculations show how much a quantity has changed relative to its original value.

Percentage Increase

Step 1: **Calculate the amount of increase:** New Value - Original Value.

Step 2: **Divide by the Original Value:** (Amount of Increase) / (Original Value).

Step 3: **Convert to Percentage:** Multiply by 100 and add %.

Example: Price increased from \$50 to \$60. Increase = $60 - 50 = 10$. Percentage Increase = $\frac{10}{50} = 0.2 = 20\%$.

Percentage Decrease

Step 1: **Calculate the amount of decrease:** Original Value - New Value.

Step 2: **Divide by the Original Value:** (Amount of Decrease) / (Original Value).

Step 3: **Convert to Percentage:** Multiply by 100 and add %.

Example: Price decreased from \$50 to \$40. Decrease = $50 - 40 = 10$. Percentage Decrease = $\frac{10}{50} = 0.2 = 20\%$.

10.6 Real-World Applications (Discounts, Sales Tax, Tips, Interest)

Percentages are used daily for financial calculations:

- **Discounts:** "25% off!" means you save 25% of the original price.
- **Sales Tax:** A percentage added to the price of goods.
- **Tips:** A percentage of the bill given to service staff.
- **Interest:** A percentage of money earned on savings or charged on loans.
- **Statistics:** Reporting data (e.g., "70% of people surveyed...").

Practice Problems

1. Convert 45% to a decimal and a fraction.
2. Convert 0.08 to a percentage and a fraction.
3. Convert $\frac{1}{5}$ to a decimal and a percentage.
4. What is 15% of 200?
5. 24 is 40% of what number?
6. A shirt originally cost \$40. It is now on sale for \$30. What is the percentage decrease?
7. You want to leave a 20% tip on a \$55 bill. How much is the tip?

Chapter 11

Measurement and Units

Measurement is how we quantify things in the world: how long, how heavy, how much liquid, how warm, etc. Units are standard amounts used to express these measurements.

11.1 Common Units (Length, Weight/Mass, Capacity, Time, Temperature, Money)

We use different units for different types of measurements:

- **Length/Distance:** Inches, feet, yards, miles (Customary); Millimeters, centimeters, meters, kilometers (Metric).
- **Weight/Mass:** Ounces, pounds, tons (Customary); Grams, kilograms (Metric).
Capacity (Liquid Volume): Fluid ounces, cups, pints, quarts, gallons (Customary); Milliliters, liters (Metric).
- **Time:** Seconds, minutes, hours, days, weeks, months, years.
- **Temperature:** Fahrenheit (Customary); Celsius (Metric).
- **Money:** Dollars, cents (or whatever currency you use).

11.2 Metric System Basics (Prefixes: Kilo, Centi, Milli)

The Metric System is used by most of the world and is based on powers of 10, making conversions very easy. The base units are the meter (for length), liter (for capacity), and gram (for mass). Prefixes indicate multiples or submultiples of the base unit:

- **Kilo- (k):** 1,000 times the base unit. (e.g., 1 kilometer = 1,000 meters)
- **Centi- (c):** $\frac{1}{100}$ (one hundredth) of the base unit. (e.g., 1 centimeter = 0.01 meters)
- **Milli- (m):** $\frac{1}{1000}$ (one thousandth) of the base unit. (e.g., 1 millimeter = 0.001 meters)

You can convert by multiplying or dividing by powers of 10, which means just moving the decimal point!

11.3 Customary System Basics (Inches, Feet, Pounds, Ounces, Gallons)

The Customary System (also known as the Imperial System) is primarily used in the United States. Its units are not based on powers of 10, so conversions require specific conversion factors. Common conversions:

- 1 foot = 12 inches
- 1 yard = 3 feet
- 1 mile = 5,280 feet
- 1 pound = 16 ounces
- 1 ton = 2,000 pounds
- 1 cup = 8 fluid ounces
- 1 pint = 2 cups
- 1 quart = 2 pints
- 1 gallon = 4 quarts

11.4 Converting Within Measurement Systems

To convert units within the same system, you multiply or divide by the appropriate conversion factor.

Step 1: ****Identify the conversion factor:**** How many of the smaller unit make up one of the larger unit?

Step 2: ****Multiply or Divide:****

- When converting from a ****larger unit to a smaller unit****, **multiply**. (You'll have more of the smaller units).
- When converting from a ****smaller unit to a larger unit****, **divide**. (You'll have fewer of the larger units).

Example (Customary): Convert 3 feet to inches. 1 foot = 12 inches. You are going from larger (feet) to smaller (inches), so multiply. $3 \text{ feet} \times 12 \text{ inches/foot} = 36 \text{ inches}$.

Example (Metric): Convert 500 centimeters to meters. 1 meter = 100 centimeters. You are going from smaller (cm) to larger (m), so divide. $500 \text{ cm} \div 100 \text{ cm/meter} = 5 \text{ meters}$.

11.5 Converting Between Metric and Customary (Simple Conversions)

Converting between systems often involves approximate values. You'll usually be given the conversion factor. Example: 1 inch $\approx$ 2.54 centimeters. Convert 10 inches to centimeters: $10 \text{ inches} \times 2.54 \text{ cm/inch} = 25.4 \text{ cm}$.

11.6 Reading Scales and Measuring Tools

Understanding how to read rulers, measuring cups, thermometers, and scales is a practical application of number lines and fractions/decimals. Pay attention to the smallest markings to determine precision.

Real-World Example:

On a ruler, if marks are every $\frac{1}{8}$ of an inch, you can read measurements to the nearest eighth. On a thermometer, if marks are every 2 degrees, you can read measurements to the nearest 2 degrees.

Practice Problems

1. What unit would you use to measure the length of a pencil? (Give both Customary and Metric options).
2. How many centimeters are in 3 meters?
3. How many ounces are in 2 pounds?
4. Convert 4.5 liters to milliliters.
5. Convert 36 inches to yards.
6. If 1 kilogram is approximately 2.2 pounds, how many pounds is 5 kilograms?
7. A recipe calls for 1.5 cups of milk. How many fluid ounces is that?

Part V

Introduction to Algebra – The Power of Variables

Welcome to Algebra! This is where math truly becomes a puzzle-solving adventure. Instead of just working with known numbers, we'll start using letters to represent unknown values. This allows us to solve problems that would be very difficult with just arithmetic.

Chapter 12

Algebraic Expressions

12.1 What Is Algebra? (Using Letters for Unknown Numbers)

Algebra is a branch of mathematics that uses letters (called variables) to represent numbers that are unknown or that can change. It's like a language where numbers are the nouns and operations (+, -, *, /) are the verbs.

Real-World Example:

Think about a simple phrase: "a number plus 5." In algebra, we can write this as $x + 5$, where x stands for "a number."

12.2 Variables and Constants

- **Variable:** A symbol, usually a letter (like x, y, a, b), that represents a number that can change or is unknown.
- **Constant:** A fixed value that does not change. These are just regular numbers.

Example: In the expression $2x + 7$:

- x is the variable.
- 2 and 7 are constants.

12.3 Terms and Coefficients

- **Term:** A single number, a single variable, or a product of numbers and variables. Terms are separated by addition or subtraction signs. Example: In $3x + 5y - 7$, the terms are $3x$, $5y$, and -7 .
- **Coefficient:** The numerical factor of a term that contains a variable. It's the number multiplied by the variable. Example: In $3x$, the coefficient is 3. In $5y$, the coefficient is 5. In x , the coefficient is understood to be 1 (since $1x = x$).

12.4 Translating Words into Algebraic Expressions

One of the key skills in algebra is to translate everyday language into mathematical expressions.

- **Addition Keywords:** sum, increased by, more than, plus, total of, added to
Example: "A number increased by 7" $\implies x + 7$
- **Subtraction Keywords:** difference, decreased by, less than, minus, subtracted from
Example: "10 less than a number" $\implies x - 10$ (Note: "less than" reverses the order!)
- **Multiplication Keywords:** product, times, multiplied by, of, twice, double
Example: "Twice a number" $\implies 2x$
- **Division Keywords:** quotient, divided by, shared equally, per
Example: "A number divided by 4" $\implies \frac{x}{4}$ or $x \div 4$

Real-World Example:

"The cost of a pizza plus a \$5 delivery fee." Let p be the cost of the pizza. Expression: $p + 5$.

12.5 Evaluating Expressions (Substituting Values)

To **evaluate** an algebraic expression means to find its numerical value when you are given a value for the variable(s).

Step 1: **Substitute:** Replace each variable with its given numerical value.

Step 2: **Calculate:** Use the order of operations (PEMDAS/BODMAS) to simplify the expression.

Example: Evaluate $3x + 7$ when $x = 4$.

- Substitute: $3(4) + 7$
- Calculate: $12 + 7 = 19$.

12.6 Combining Like Terms

Like terms are terms that have the exact same variable parts (same variables raised to the same powers). Constants are also like terms. You can add or subtract like terms by combining their coefficients.

Real-World Example:

Think of "apples" and "bananas." You can combine 3 apples and 2 apples to get 5 apples, but you can't combine 3 apples and 2 bananas to get 5 "apple-bananas." In algebra, $3x$ and $2x$ are like terms (like apples), but $3x$ and $2y$ are not (like apples and bananas).

Step 1: **Identify like terms:** Look for terms with identical variable parts.

Step 2: **Combine coefficients:** Add or subtract the coefficients of the like terms. Keep the variable part the same.

Example: Simplify $5x + 3y - 2x + y + 8$

- Like terms with x : $5x$ and $-2x$. Combine: $5x - 2x = 3x$.
- Like terms with y : $3y$ and y (which is $1y$). Combine: $3y + y = 4y$.
- Constant: 8.
- Result: $3x + 4y + 8$.

12.7 Distributive Property

The **distributive property** allows you to multiply a sum or difference by a number. It states that $a(b + c) = ab + ac$.

Step 1: **Multiply the outside number by EACH term inside the parentheses.**

Example: $4(x + 3)$ $4 \times x + 4 \times 3 = 4x + 12$.

Example: $2(y - 5)$ $2 \times y - 2 \times 5 = 2y - 10$.

Practice Problems

1. In the expression $7a - 3b + 12$:
 - Name the variables.
 - Name the constants.
 - Name the coefficients.
 - How many terms are there?
2. Translate into an algebraic expression:
 - "A number decreased by 9"
 - "The product of 5 and a number"
 - "7 more than half a number"
3. Evaluate the expression $2x^2 + 5x - 1$ when $x = 3$.
4. Simplify by combining like terms: $8p + 3q - 5p + 2q$
5. Simplify by combining like terms: $10 - 4m + 7 + 2m$
6. Apply the distributive property: $6(y + 4)$
7. Apply the distributive property: $3(2x - 7)$

Chapter 13

Solving Basic Equations

An **equation** is a mathematical statement that shows two expressions are equal. It always contains an equals sign ($=$). Our goal in solving an equation is to find the value of the variable that makes the equation true.

13.1 What Is an Equation? (Balanced Scale)

Think of an equation as a perfectly balanced scale. Whatever you do to one side of the scale, you must do to the other side to keep it balanced. Example: $x + 3 = 7$ The left side ($x + 3$) must have the same value as the right side (7).

Why is this important?

The key principle of solving equations: **Whatever you do to one side of the equation, you must do to the other side.** This keeps the equation balanced and the equality true.

13.2 Solving One-Step Linear Equations (+, -, *, /)

A "one-step" equation means you only need to perform one operation to isolate the variable (get it by itself). To isolate the variable, you use the **inverse operation**.

- The inverse of addition is subtraction.
- The inverse of subtraction is addition.
- The inverse of multiplication is division.
- The inverse of division is multiplication.

Solving One-Step Addition/Subtraction Equations

Example: Solve $x + 5 = 12$

Step 1: **Identify the operation:** x is being added to 5.

Step 2: **Use inverse operation:** Subtract 5 from both sides. $x + 5 - 5 = 12 - 5$

Step 3: **Simplify:** $x = 7$

Check your answer: $7 + 5 = 12$. (True!)

Example: Solve $y - 3 = 8$

Step 1: **Identify the operation:** y is being subtracted by 3.

Step 2: **Use inverse operation:** Add 3 to both sides. $y - 3 + 3 = 8 + 3$

Step 3: **Simplify:** $y = 11$

Check your answer: $11 - 3 = 8$. (True!)

Solving One-Step Multiplication/Division Equations

Example: Solve $4x = 20$ (Remember $4x$ means $4 \times x$)

Step 1: **Identify the operation:** x is being multiplied by 4.

Step 2: **Use inverse operation:** Divide both sides by 4. $\frac{4x}{4} = \frac{20}{4}$

Step 3: **Simplify:** $x = 5$

Check your answer: $4 \times 5 = 20$. (True!)

Example: Solve $\frac{m}{3} = 7$ (Remember $\frac{m}{3}$ means $m \div 3$)

Step 1: **Identify the operation:** m is being divided by 3.

Step 2: **Use inverse operation:** Multiply both sides by 3. $\frac{m}{3} \times 3 = 7 \times 3$

Step 3: **Simplify:** $m = 21$

Check your answer: $\frac{21}{3} = 7$. (True!)

13.3 Solving Two-Step Linear Equations

A two-step equation requires two inverse operations to isolate the variable. Generally, you'll undo addition/subtraction first, then multiplication/division.

Example: Solve $2x + 3 = 11$

Step 1: **Undo addition/subtraction:** Subtract 3 from both sides. $2x + 3 - 3 = 11 - 3$
 $2x = 8$

Step 2: **Undo multiplication/division:** Divide both sides by 2. $\frac{2x}{2} = \frac{8}{2}$ $x = 4$

Check: $2(4) + 3 = 8 + 3 = 11$. (True!)

Example: Solve $\frac{n}{4} - 2 = 5$

Step 1: **Undo addition/subtraction:** Add 2 to both sides. $\frac{n}{4} - 2 + 2 = 5 + 2$ $\frac{n}{4} = 7$

Step 2: **Undo multiplication/division:** Multiply both sides by 4. $\frac{n}{4} \times 4 = 7 \times 4$
 $n = 28$

Check: $\frac{28}{4} - 2 = 7 - 2 = 5$. (True!)

13.4 Solving Equations with Variables on Both Sides (Basic)

When variables are on both sides of the equation, your first step is to gather all the variable terms on one side and the constant terms on the other.

Example: Solve $5x - 7 = 2x + 8$

Step 1: ****Move variable terms to one side:**** Subtract $2x$ from both sides. $5x - 2x - 7 = 2x - 2x + 8$ $3x - 7 = 8$

Step 2: ****Move constant terms to the other side:**** Add 7 to both sides. $3x - 7 + 7 = 8 + 7$ $3x = 15$

Step 3: ****Isolate the variable:**** Divide both sides by 3. $\frac{3x}{3} = \frac{15}{3}$ $x = 5$

Check: $5(5) - 7 = 25 - 7 = 18$. And $2(5) + 8 = 10 + 8 = 18$. (Both sides are 18, so true!)

13.5 Word Problems Leading to Linear Equations

Algebra is powerful for solving word problems!

Step 1: ****Read and Define:**** Read the problem carefully. Identify what you need to find and assign a variable (e.g., let x be the unknown number).

Step 2: ****Translate:**** Write an equation that represents the situation described in the problem. Look for keywords that indicate operations or equality ("is," "equals").

Step 3: ****Solve:**** Solve the equation using the inverse operations.

Step 4: ****Check and State:**** Check your answer in the original word problem. Write your answer in a complete sentence with units.

Real-World Example:

"Sarah bought a book for \$15 and some pens. She spent a total of \$27. If each pen costs \$2, how many pens did she buy?"

- Define: Let p = number of pens.
- Translate: Cost of book + cost of pens = total spent $15 + 2p = 27$
- Solve: $15 + 2p - 15 = 27 - 15$ $2p = 12$ $\frac{2p}{2} = \frac{12}{2}$ $p = 6$
- Check: $15 + 2(6) = 15 + 12 = 27$. (Correct!)
- State: Sarah bought 6 pens.

Practice Problems

1. Solve for x : $x + 10 = 25$
2. Solve for y : $y - 7 = 15$
3. Solve for a : $6a = 42$

4. Solve for b : $\frac{b}{5} = 10$
5. Solve for m : $3m - 4 = 17$
6. Solve for n : $\frac{n}{2} + 5 = 9$
7. Solve for p : $7p - 1 = 4p + 11$
8. Twice a number, increased by 5, is 19. What is the number? (Set up and solve an equation)
9. A rectangle has a perimeter of 30 cm. Its length is 10 cm. What is its width? (Hint: Perimeter = $2L + 2W$)

Chapter 14

Inequalities and Graphing

Sometimes, we don't just want to know when things are *equal*, but when they are *greater than*, *less than*, or *at least* some amount. This is where inequalities come in handy!

14.1 What Are Inequalities? ($>$, $<$, $\geq$, $\leq$)

An **inequality** is a mathematical statement that compares two expressions using an inequality symbol. It means the two sides are *not necessarily equal*.

- $<$ (Less than): Example: $x < 5$ (x is any number smaller than 5)
- $>$ (Greater than): Example: $x > 5$ (x is any number larger than 5)
- $\leq$ (Less than or equal to): Example: $x \leq 5$ (x is 5 or any number smaller than 5)
- $\geq$ (Greater than or equal to): Example: $x \geq 5$ (x is 5 or any number larger than 5)
- $\neq$ (Not equal to): Example: $x \neq 5$ (x is any number except 5)

Real-World Example:

- You must be over 18 to vote: age > 18 .
- You need at least \$10 to buy the movie ticket: money ≥ 10 .
- The car speed must be less than or equal to 60 mph: speed ≤ 60 .

14.2 Graphing Inequalities on a Number Line

We can show the solutions to inequalities on a number line.

- Use an **open circle** (o) for $<$ or $>$ (the number itself is NOT included).
- Use a **closed circle** (•) for $\leq$ or $\geq$ (the number itself IS included).
- Draw an arrow to show all the numbers that satisfy the inequality.

Examples:

- $x > 3$: Open circle at 3, arrow pointing right.
- $x \leq -1$: Closed circle at -1, arrow pointing left.
- $x \geq 0$: Closed circle at 0, arrow pointing right.
- $x < 2$: Open circle at 2, arrow pointing left.

(In a real book, you'd show diagrams of these number lines.)

14.3 Solving One-Step Inequalities

Solving inequalities is very similar to solving equations, with one crucial difference! You use inverse operations to isolate the variable.

The Special Rule: When you multiply or divide both sides of an inequality by a **negative number**, you must **reverse the inequality sign**.

Adding/Subtracting

Example: Solve $x + 4 > 9$ Subtract 4 from both sides: $x + 4 - 4 > 9 - 4 \implies x > 5$. (Graph: open circle at 5, arrow right)

Example: Solve $y - 2 \leq 1$ Add 2 to both sides: $y - 2 + 2 \leq 1 + 2 \implies y \leq 3$. (Graph: closed circle at 3, arrow left)

Multiplying/Dividing (Positive number)

Example: Solve $3x < 12$ Divide by 3 (positive): $\frac{3x}{3} < \frac{12}{3} \implies x < 4$. (Graph: open circle at 4, arrow left)

Multiplying/Dividing (Negative number)

Example: Solve $-2x \geq 6$ Divide by -2 (negative!): $\frac{-2x}{-2} \leq \frac{6}{-2}$ (Reverse the sign!) $\implies x \leq -3$. (Graph: closed circle at -3, arrow left)

14.4 Introduction to the Coordinate Plane (x-y Axis, Plotting Points)

The **coordinate plane** (also called the Cartesian plane) is a two-dimensional surface where we can locate points using ordered pairs of numbers. It's formed by two perpendicular number lines:

- The **x-axis**: The horizontal number line.
- The **y-axis**: The vertical number line.
- The **origin**: The point where the x-axis and y-axis intersect (0,0).

Points are located using **ordered pairs** (x, y) , where x tells you how far to move horizontally from the origin, and y tells you how far to move vertically.

Step 1: **Start at the origin (0,0).**

Step 2: **Move horizontally (x-coordinate):** Move right for positive x , left for negative x .

Step 3: **Move vertically (y-coordinate):** Move up for positive y , down for negative y .

Step 4: **Place your point.**

Example: Plot the point (3, 2). Start at (0,0). Move 3 units right. Then move 2 units up. Place the point.

Real-World Example:

The coordinate plane is used in maps (latitude and longitude), computer graphics, video games, and showing relationships between two different quantities (like time vs. distance).

14.5 Graphing Simple Linear Equations (e.g., $y = x + 2$)

A **linear equation** is an equation whose graph is a straight line. The simplest way to graph them is by plotting points.

Step 1: **Choose a few values for x : Pick a few easy numbers, like -1, 0, 1, 2.

Step 2: **Substitute each x -value into the equation** to find the corresponding y -value.

Step 3: **Create ordered pairs (x , y):** These are your points.

Step 4: **Plot the points** on the coordinate plane.

Step 5: **Draw a straight line** through the points. Extend the line with arrows to show it continues infinitely.

Example: Graph the equation $y = x + 2$

- If $x = -1$, $y = -1 + 2 = 1$. Point: $(-1, 1)$
- If $x = 0$, $y = 0 + 2 = 2$. Point: $(0, 2)$
- If $x = 1$, $y = 1 + 2 = 3$. Point: $(1, 3)$
- If $x = 2$, $y = 2 + 2 = 4$. Point: $(2, 4)$

Plot these points and connect them with a straight line.

Practice Problems

1. Write the inequality for "You need to be at most 16 years old to get a junior driver's license."
2. Graph $x > 4$ on a number line.
3. Graph $x \leq -2$ on a number line.

4. Solve and graph on a number line: $x + 3 < 7$
5. Solve and graph on a number line: $5y \geq 15$
6. Solve and graph on a number line: $-2m < 10$
7. Plot the following points on a coordinate plane: A(4, 1), B(-2, 3), C(0, -4), D(-1, -1).
8. Create a table of values and graph the linear equation $y = 2x - 1$.

Part VI

Basic Geometry – Shapes, Space, and Measurement

Geometry is the study of shapes, sizes, positions, and properties of space. It's about understanding the world around us visually and mathematically. From the houses we live in to the games we play, geometry is everywhere!

Chapter 15

Fundamental Geometric Concepts

15.1 Points, Lines, Rays, Line Segments

These are the most basic building blocks of geometry.

- **Point:** An exact location in space. It has no size, only position. We represent it with a dot and a capital letter (e.g., Point A).
- **Line:** A perfectly straight path that extends infinitely in both directions. It has no thickness. We name a line by two points on it (e.g., Line AB or $\overleftrightarrow{AB}$) or a lowercase letter (e.g., line l).
- **Ray:** A part of a line that has one endpoint and extends infinitely in one direction. We name it starting with the endpoint (e.g., Ray AB or $\overrightarrow{AB}$).
- **Line Segment:** A part of a line that has two endpoints. It has a definite length. We name it by its two endpoints (e.g., Line Segment AB or $\overline{AB}$).

Real-World Example:

- A tiny dot on a map is a **point**.
- A laser beam is like a **ray**.
- The edge of a table is a **line segment**.

15.2 Angles (Acute, Obtuse, Right, Straight, Reflex) and Their Measurement

An **angle** is formed when two rays share a common endpoint. The common endpoint is called the **vertex**, and the rays are called the **sides** or arms of the angle. We measure angles in **degrees** ($^\circ$). A full circle is 360° .

- **Right Angle:** Exactly 90° . It looks like a perfect corner (like a square corner of a book). Often marked with a small square.
- **Acute Angle:** Less than 90° . It's a "sharp" angle.
- **Obtuse Angle:** Greater than 90° but less than 180° . It's a "wide" angle.

- **Straight Angle:** Exactly 180° . It forms a straight line.
- **Reflex Angle:** Greater than 180° but less than 360° .

15.3 Types of Lines (Parallel, Perpendicular, Intersecting)

- **Intersecting Lines:** Lines that cross each other at exactly one point.
- **Perpendicular Lines:** Intersecting lines that form a right angle (90°) at their intersection. Symbol: $\perp$ (e.g., Line $m \perp$ Line n)
- **Parallel Lines:** Lines that are always the same distance apart and never intersect, no matter how far they are extended. Symbol: $\parallel$ (e.g., Line $p \parallel$ Line q)

Real-World Example:

- The corner of a wall where two walls meet forms a **right angle**.
- Railroad tracks are **parallel lines**.
- A crossroad where streets meet at a perfect right angle forms **perpendicular lines**.

15.4 Polygons (Triangle, Quadrilateral, Pentagon, Hexagon, Octagon)

A **polygon** is a closed 2D shape made up of three or more straight line segments (sides) connected end-to-end. The sides only meet at their endpoints, called vertices.

- **Triangle:** 3 sides, 3 vertices.
- **Quadrilateral:** 4 sides, 4 vertices (e.g., square, rectangle, rhombus, trapezoid).
- **Pentagon:** 5 sides, 5 vertices.
- **Hexagon:** 6 sides, 6 vertices.
- **Octagon:** 8 sides, 8 vertices.

15.5 Circles (Radius, Diameter, Circumference – Intro to Pi)

A **circle** is a set of all points in a plane that are the same distance from a central point.

- **Center:** The central point from which all points on the circle are equidistant.
- **Radius (r):** The distance from the center to any point on the circle.

- **Diameter (d):** The distance across the circle through its center. The diameter is always twice the radius ($d = 2r$).
- **Circumference (C):** The distance around the circle (its perimeter).

Introduction to Pi (π): For any circle, if you divide its circumference by its diameter, you always get the same number. This special number is called **Pi**, represented by the Greek letter π . $\pi \approx 3.14159$ (often approximated as 3.14 or $\frac{22}{7}$). So, the formula for circumference is $C = \pi \times d$ or $C = 2 \times \pi \times r$.

Real-World Example:

- A bicycle wheel is a circle. The spoke is like the **radius**. The distance across the wheel is the **diameter**. The length of the tire is the **circumference**.

Practice Problems

1. Draw and label: a line segment XY, a ray AB.
2. What type of angle is 45° ? What type is 135° ?
3. If two lines never meet, what are they called?
4. How many sides does a hexagon have?
5. If a circle has a radius of 5 cm, what is its diameter?
6. If a circle has a diameter of 10 meters, what is its circumference (use $\pi \approx 3.14$)?

Chapter 16

Perimeter, Area, and Volume

These concepts help us measure the "size" of shapes and objects in terms of their boundaries, surfaces, or the space they occupy.

16.1 Perimeter of Polygons (Square, Rectangle, Triangle)

Perimeter is the total distance around the outside edge of a 2D shape. To find the perimeter, you add up the lengths of all its sides.

Real-World Example:

If you want to put a fence around your garden, you need to know its perimeter.

- **Square:** All 4 sides are equal. If side length is s , Perimeter $P = s + s + s + s = 4s$. Example: A square with side 6 cm. $P = 4 \times 6 = 24$ cm.
- **Rectangle:** Opposite sides are equal. If length is L and width is W , Perimeter $P = L + W + L + W = 2L + 2W$. Example: A rectangle with length 8 ft and width 3 ft. $P = 2(8) + 2(3) = 16 + 6 = 22$ ft.
- **Triangle:** Add the lengths of its three sides. If sides are a, b, c , Perimeter $P = a + b + c$. Example: A triangle with sides 5 cm, 7 cm, and 9 cm. $P = 5 + 7 + 9 = 21$ cm.

Units for perimeter are linear units (e.g., cm, ft, m).

16.2 Area of Polygons (Square, Rectangle, Triangle, Parallelogram)

Area is the amount of flat surface (2D space) a shape covers. It's measured in square units (e.g., cm^2 , ft^2 , m^2).

Real-World Example:

If you want to buy carpet for your living room, you need to know its area.

- **Square:** Area $A = \text{side} \times \text{side} = s^2$. Example: A square with side 6 cm. $A = 6 \times 6 = 36 \text{ cm}^2$.

- **Rectangle:** Area $A = \text{length} \times \text{width} = L \times W$. Example: A rectangle with length 8 ft and width 3 ft. $A = 8 \times 3 = 24 \text{ ft}^2$.
- **Triangle:** Area $A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh$. The base and height must be perpendicular. Example: A triangle with base 10 cm and height 4 cm. $A = \frac{1}{2} \times 10 \times 4 = 20 \text{ cm}^2$.
- **Parallelogram:** Area $A = \text{base} \times \text{height} = bh$. The base and height must be perpendicular. Example: A parallelogram with base 7 inches and height 5 inches. $A = 7 \times 5 = 35 \text{ in}^2$.

16.3 Area of a Circle (Basic Formula)

The area of a circle is $A = \pi \times \text{radius}^2 = \pi r^2$. Example: A circle with radius 5 cm. $A = \pi \times 5^2 = 25\pi \text{ cm}^2$. (Using $\pi \approx 3.14$, $A \approx 25 \times 3.14 = 78.5 \text{ cm}^2$).

16.4 Introduction to 3D Shapes (Cube, Rectangular Prism, Cylinder, Sphere, Pyramid, Cone)

These are shapes that have three dimensions: length, width, and height/depth. They occupy space.

- **Cube:** A 3D shape with 6 square faces (like a dice).
- **Rectangular Prism:** A 3D shape with 6 rectangular faces (like a brick or box).
- **Cylinder:** A 3D shape with two parallel circular bases connected by a curved surface (like a can).
- **Sphere:** A perfectly round 3D object where every point on its surface is equidistant from its center (like a ball).
- **Pyramid:** A 3D shape with a polygon base and triangular faces that meet at a common vertex (apex).
- **Cone:** A 3D shape with a circular base and a curved surface that tapers to a single point (apex).

16.5 Volume of Rectangular Prisms and Cubes

Volume is the amount of space a 3D object occupies. It's measured in cubic units (e.g., cm^3 , ft^3 , m^3).

Real-World Example:

If you want to know how much water can fill a swimming pool, you need to calculate its volume.

- **Rectangular Prism:** Volume $V = \text{length} \times \text{width} \times \text{height} = L \times W \times H$. Example: A rectangular prism 5 cm long, 3 cm wide, and 2 cm high. $V = 5 \times 3 \times 2 = 30 \text{ cm}^3$.
- **Cube:** Since all sides are equal (s), Volume $V = s \times s \times s = s^3$. Example: A cube with side 4 inches. $V = 4 \times 4 \times 4 = 64 \text{ in}^3$.

Practice Problems

1. A square garden has sides of 12 meters. What is its perimeter and area?
2. A rectangular swimming pool is 25 feet long and 10 feet wide. What is the area of the water surface?
3. Find the perimeter of a triangle with sides 8 cm, 15 cm, and 17 cm.
4. Calculate the area of a triangle with a base of 6 inches and a height of 8 inches.
5. A circular mat has a radius of 7 cm. What is its area? (Use $\pi \approx \frac{22}{7}$)
6. How many cubic meters of concrete are needed to fill a rectangular foundation that is 10 m long, 5 m wide, and 2 m deep?

Chapter 17

Symmetry, Transformations, and Basic Pythagorean Theorem

17.1 Line Symmetry and Rotational Symmetry

- **Line Symmetry:** A shape has line symmetry if it can be folded along a line (the line of symmetry) so that the two halves match exactly. Example: A square has 4 lines of symmetry. A heart shape has 1.
- **Rotational Symmetry:** A shape has rotational symmetry if it looks the same after being rotated less than a full 360° turn around a central point. The order of rotational symmetry is how many times the shape looks identical during a full turn. Example: A square has rotational symmetry of order 4 (it looks the same after 90° , 180° , 270° turns).

17.2 Transformations (Translation/Slide, Reflection/Flip, Rotation/Turn)

Transformations are ways to move geometric shapes without changing their size or shape.

- **Translation (Slide):** Moving a shape from one place to another without rotating or flipping it. Every point moves the same distance in the same direction.
- **Reflection (Flip):** Flipping a shape over a line (the line of reflection) to create a mirror image.
- **Rotation (Turn):** Turning a shape around a fixed point (the center of rotation) by a certain angle.

17.3 Introduction to the Pythagorean Theorem (Finding Missing Sides of Right Triangles)

The **Pythagorean Theorem** is a fundamental rule in geometry that applies **ONLY** to **right triangles** (triangles with one 90° angle). It describes the relationship between the lengths of the sides of a right triangle.

- The two shorter sides that form the right angle are called **legs** (often labeled a and b).
- The longest side, opposite the right angle, is called the **hypotenuse** (always labeled c).

Theorem: The square of the length of the hypotenuse (c) is equal to the sum of the squares of the lengths of the two legs (a and b). Formula: $a^2 + b^2 = c^2$

Step 1: ****Identify the legs and hypotenuse:**** Make sure you know which sides are a , b , and c .

Step 2: ****Substitute known values:**** Plug the lengths of the known sides into the formula.

Step 3: ****Solve for the unknown side:**** Use algebraic techniques (squaring, adding, subtracting, square rooting) to find the missing length.

Real-World Example:

You have a ladder that is 10 feet long (c). You place its base 6 feet away from a wall (a). How high up the wall (b) will the ladder reach?

- $a^2 + b^2 = c^2$
- $6^2 + b^2 = 10^2$
- $36 + b^2 = 100$
- $b^2 = 100 - 36$
- $b^2 = 64$
- $b = \sqrt{64}$
- $b = 8$

The ladder reaches 8 feet up the wall.

Practice Problems

1. Does the letter 'E' have line symmetry? If so, how many lines of symmetry?
2. Does a regular pentagon have rotational symmetry? If so, what is its order?
3. Describe the transformation if a shape is moved straight up 3 units and 2 units to the left.
4. A triangle has sides of length 3, 4, and 5. Is it a right triangle? Show your work.
5. A right triangle has legs measuring 5 cm and 12 cm. What is the length of its hypotenuse?
6. If the hypotenuse of a right triangle is 13 inches and one leg is 5 inches, what is the length of the other leg?

Part VII

Data and Probability – Making Sense of Information

In our modern world, we are surrounded by information, or "data." This part of the book will teach you how to collect, organize, display, and understand data, as well as how to think about chance and likelihood.

Chapter 18

Collecting and Representing Data

18.1 What Is Data?

Data is information, often in the form of numbers or facts, that can be collected, analyzed, and used to make decisions or draw conclusions.

Real-World Example:

- The number of students who prefer chocolate ice cream.
- The different types of pets owned by families on your street.
- The daily high temperature for a month.

All these are examples of data.

18.2 Methods of Collecting Data (Surveys, Observation)

How do we get data?

- **Surveys/Questionnaires:** Asking people questions directly (e.g., "What is your favorite color?").
- **Observation:** Watching and recording information without directly interacting (e.g., counting how many red cars pass by in an hour).
- **Experiments:** Setting up a controlled situation to see what happens (e.g., testing how different amounts of fertilizer affect plant growth).
- **Existing Records:** Using information that has already been collected (e.g., looking up historical weather data).

18.3 Organizing Data (Tally Charts, Frequency Tables)

Once you collect data, it often looks messy. Organizing it makes it easier to understand.

- **Tally Chart:** Uses tally marks (strokes) to count occurrences of each item. Each fifth mark is drawn diagonally across the previous four (||||). Example: Favorite

	Fruit	Tally	Frequency
Fruits	Apple	II	7
	Banana		5
	Orange		3

- **Frequency Table:** A table that lists categories of data and the number of times each category appears (its frequency). The "Frequency" column in the tally chart above is a frequency table.

18.4 Types of Graphs: Bar Graphs, Pictographs, Line Graphs, Circle Graphs (Pie Charts)

Graphs are visual ways to represent data, making patterns and comparisons easy to see.

- **Bar Graph:** Uses bars to show the frequency of different categories. The longer the bar, the greater the frequency.
 - Good for comparing discrete categories.
 - Has a title, labeled axes, and consistent scales.
- **Pictograph:** Uses pictures or symbols to represent data. A key explains what each symbol represents.
 - Easy to understand at a glance.
 - Can be tricky if partial symbols are needed.
- **Line Graph:** Uses points connected by lines to show how data changes over time.
 - Excellent for showing trends (increases, decreases, stability).
 - X-axis usually represents time.
- **Circle Graph (Pie Chart):** A circle divided into sectors (like slices of a pie), where each sector represents a proportion of the whole. The sum of all percentages in a pie chart must be 100%.
 - Good for showing parts of a whole.
 - Not ideal for comparing precise quantities or showing changes over time.

(In a real book, you would include simple examples of each graph type.)

18.5 Reading and Interpreting Graphs

The most important part of working with data is being able to understand what the graph is telling you.

Step 1: **Read the Title:** What is the graph about?

Step 2: ****Look at the Labels:**** What do the axes (for bar/line graphs) or categories (for pie charts) represent? What are the units?

Step 3: ****Check the Scale:**** How are the numbers counted on the axes? Is the scale consistent?

Step 4: ****Analyze the Data:**** Look for patterns, trends, highest/lowest values, and comparisons.

Step 5: ****Draw Conclusions:**** What can you say about the data based on the graph?

Real-World Example:

Look at a line graph showing a company's sales over 12 months.

- You can see if sales are generally increasing or decreasing.
- Identify peak sales months and lowest sales months.
- Compare sales from one quarter to another.

Practice Problems

1. You are collecting data on the colors of cars in a parking lot. How would you record this data using a tally chart? (List 3 colors with example tallies).
2. Create a simple bar graph from the following data:
 - Apples: 5
 - Oranges: 3
 - Bananas: 7(Describe your graph's axes and bars.)
3. What type of graph would be best to show how a student's height has changed over the past 5 years? Why?
4. A pie chart shows that 30% of students walk to school, 50% ride the bus, and 20% are driven. If there are 200 students, how many walk to school?

Chapter 19

Analyzing Data – Mean, Median, Mode, Range

Once data is collected and organized, we often want to summarize it with a single number or a few numbers that describe its main characteristics. These are called measures of central tendency and spread.

19.1 Mean (Average)

The **mean** is what most people think of as the "average." It's found by adding up all the values in a data set and then dividing by the number of values.

Step 1: ****Sum the values:**** Add all the numbers in your data set.

Step 2: ****Count the values:**** Count how many numbers are in your data set.

Step 3: ****Divide:**** Divide the sum (Step 1) by the count (Step 2).

Example: Find the mean of the test scores: 85, 90, 78, 92, 85. $\text{Sum} = 85 + 90 + 78 + 92 + 85 = 430$. $\text{Count} = 5$ scores. $\text{Mean} = \frac{430}{5} = 86$.

19.2 Median (Middle Value)

The **median** is the middle value in a data set when the values are arranged in order from least to greatest. It's helpful because it's not affected by extremely high or low values (outliers).

Step 1: ****Order the data:**** Arrange all numbers from smallest to largest.

Step 2: ****Find the middle:****

- If there is an ****odd**** number of values, the median is the single middle number.
- If there is an ****even**** number of values, the median is the average (mean) of the two middle numbers.

Example (Odd count): Find the median of: 85, 90, 78, 92, 85. Order: 78, 85, **85**, 90, 92. The median is 85.

Example (Even count): Find the median of: 10, 15, 20, 25. Order: 10, **15, 20**, 25. The two middle numbers are 15 and 20. Median = $\frac{15+20}{2} = \frac{35}{2} = 17.5$.

19.3 Mode (Most Frequent Value)

The **mode** is the value that appears most frequently in a data set. A data set can have one mode, multiple modes, or no mode at all.

Step 1: ****Count occurrences:**** Go through your data and count how many times each value appears.

Step 2: ****Identify most frequent:**** The value(s) with the highest count is/are the mode(s).

Example: Find the mode of: 85, 90, 78, 92, 85. 85 appears twice. All other numbers appear once. The mode is 85.

Example: Find the mode of: 1, 2, 2, 3, 3, 4. 2 appears twice, 3 appears twice. The modes are 2 and 3 (bimodal).

Example: Find the mode of: 5, 10, 15, 20. No number repeats. There is no mode.

19.4 Range (Spread of Data)

The **range** is a measure of the spread or variability of a data set. It's the difference between the highest value and the lowest value.

Step 1: ****Find highest and lowest:**** Identify the largest and smallest numbers in the data set.

Step 2: ****Subtract:**** Subtract the lowest value from the highest value.

Example: Find the range of: 85, 90, 78, 92, 85. Highest value = 92. Lowest value = 78. Range = $92 - 78 = 14$.

19.5 Choosing the Best Measure of Central Tendency

- **Mean** is best when the data is fairly symmetrical and doesn't have extreme outliers.
- **Median** is best when the data has extreme high or low values (outliers) that might skew the mean (e.g., typical house prices in an area).
- **Mode** is best for categorical data (e.g., favorite color, most popular shoe size) or when you want to know the most common item.

Practice Problems

1. Find the mean, median, mode, and range for the following set of numbers representing daily temperatures (in degrees Celsius): 18, 22, 20, 25, 18, 23.
2. A student's quiz scores are: 7, 8, 9, 10, 6, 8, 7, 9, 8, 10. Find the mean, median, mode, and range.

3. Why might the median salary be a better representation of "average" salary than the mean in a company where the CEO earns significantly more than everyone else?

Chapter 20

Introduction to Probability

Life is full of uncertainties! Probability is the branch of math that helps us measure the likelihood of events happening.

20.1 What Is Probability? (Likelihood of an Event)

Probability is a measure of how likely an event is to occur. It's expressed as a number between 0 and 1 (or 0% and 100%).

- Probability of 0 (or 0%): The event is impossible.
- Probability of 1 (or 100%): The event is certain to happen.
- Probability of 0.5 (or 50%): The event is equally likely to happen or not happen.

20.2 Outcomes and Events

- **Outcome:** A single possible result of an experiment or situation. Example: When you flip a coin, the outcomes are Heads or Tails.
- **Event:** A set of one or more outcomes. Example: Flipping a coin and getting Heads. Rolling a die and getting an even number (outcomes: 2, 4, 6).

20.3 Calculating Simple Probability ($P = \text{Favorable Outcomes} / \text{Total Outcomes}$)

For situations where all outcomes are equally likely, the probability of an event is calculated as: $P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$

Real-World Example:

What is the probability of rolling a 4 on a standard six-sided die?

- Favorable outcomes (rolling a 4): 1 (only the number 4)
- Total possible outcomes (rolling any side): 6 (1, 2, 3, 4, 5, 6)
- $P(\text{rolling a 4}) = \frac{1}{6}$.

Real-World Example:

What is the probability of picking a red marble from a bag containing 3 red marbles and 5 blue marbles?

- Favorable outcomes (red marbles): 3
- Total possible outcomes (all marbles): $3 + 5 = 8$
- $P(\text{picking red}) = \frac{3}{8}$.

20.4 Certain, Impossible, Likely, Unlikely, Equally Likely Events

We can describe the likelihood of events using these terms:

- **Impossible:** Will never happen. Probability = 0. (e.g., rolling a 7 on a standard 6-sided die).
- **Unlikely:** Has a low chance of happening. Probability is close to 0 but greater than 0. (e.g., winning the lottery).
- **Equally Likely/As Likely as Not:** Has a 50% chance of happening. Probability = 0.5. (e.g., flipping a coin and getting heads).
- **Likely:** Has a high chance of happening. Probability is close to 1 but less than 1. (e.g., the sun rising tomorrow).
- **Certain:** Will definitely happen. Probability = 1. (e.g., the day following Tuesday will be Wednesday).

20.5 Simple Experiments (Coin Flips, Dice Rolls)

- ****Coin Flip:**** 2 outcomes (Heads, Tails). $P(\text{Heads}) = \frac{1}{2}$, $P(\text{Tails}) = \frac{1}{2}$.
- ****Die Roll:**** 6 outcomes (1, 2, 3, 4, 5, 6). $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$ (outcomes: 2, 4, 6). $P(\text{number greater than 4}) = \frac{2}{6} = \frac{1}{3}$ (outcomes: 5, 6).

Why is this important?

Probability helps us understand risks, make predictions, and play games fairly.

Practice Problems

1. A spinner has 4 equal sections: Red, Blue, Green, Yellow.
 - What is the probability of landing on Red?
 - What is the probability of landing on a primary color (Red, Blue, Yellow)?
 - Describe the probability of landing on Orange using words (impossible, likely, etc.).

2. You have a deck of 52 playing cards.
 - What is the probability of drawing an Ace?
 - What is the probability of drawing a red card?
3. If the probability of rain is 0.7, what is the probability that it will NOT rain?

Part VIII

Stepping Stones to Advanced Math – Logic, Exponents, and Functions

This final part introduces concepts that will be foundational for more advanced mathematics, such as algebra beyond basic equations, calculus, and computer science.

Chapter 21

Exponents and Roots

You've had a brief introduction to exponents with order of operations. Now, let's dive deeper!

21.1 What Are Exponents? (Powers, Bases)

An **exponent** (or power) is a shorthand way to write repeated multiplication of the same number. In a^n :

- a is the **base** (the number being multiplied).
- n is the **exponent** (how many times the base is used as a factor).

Example: 2^3 means $2 \times 2 \times 2 = 8$. (Read as "2 to the power of 3" or "2 cubed"). Example: 5^2 means $5 \times 5 = 25$. (Read as "5 to the power of 2" or "5 squared").

Why is this important?

Don't confuse exponents with multiplication: 2^3 is not 2×3 .

21.2 Basic Exponent Rules (Multiplication, Division with Same Base)

These rules simplify calculations when working with exponents.

Multiplication Rule (Same Base)

When multiplying powers with the same base, **add the exponents**. Formula: $a^m \times a^n = a^{m+n}$ Example: $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5 = 32$. Using the rule: $2^{3+2} = 2^5 = 32$.

Division Rule (Same Base)

When dividing powers with the same base, **subtract the exponents**. Formula: $\frac{a^m}{a^n} = a^{m-n}$ (where $a \neq 0$) Example: $\frac{2^5}{2^2} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2 \times 2 \times 2 = 2^3 = 8$. Using the rule: $2^{5-2} = 2^3 = 8$.

Power of a Power Rule

When raising a power to another power, **multiply the exponents**. Formula: $(a^m)^n = a^{m \times n}$ Example: $(2^3)^2 = (2 \times 2 \times 2)^2 = (8)^2 = 8 \times 8 = 64$. Using the rule: $2^{3 \times 2} = 2^6 = 64$.

21.3 Negative Exponents (Introduction)

A negative exponent means to take the **reciprocal** of the base raised to the positive exponent. Formula: $a^{-n} = \frac{1}{a^n}$ (where $a \neq 0$) Example: $2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$.

Why is this important?

Any non-zero number raised to the power of zero is 1. (e.g., $5^0 = 1, x^0 = 1$).

21.4 Square Roots and Cube Roots (Perfect Squares/Cubes)

Square Roots

A **square root** of a number is a value that, when multiplied by itself, gives the original number. The symbol is $\sqrt{\quad}$. Example: $\sqrt{25} = 5$ because $5 \times 5 = 25$. Example: $\sqrt{100} = 10$ because $10 \times 10 = 100$. Numbers like 1, 4, 9, 16, 25... are called **perfect squares** because their square roots are whole numbers.

Cube Roots

A **cube root** of a number is a value that, when multiplied by itself three times, gives the original number. The symbol is $\sqrt[3]{\quad}$. Example: $\sqrt[3]{8} = 2$ because $2 \times 2 \times 2 = 8$. Example: $\sqrt[3]{27} = 3$ because $3 \times 3 \times 3 = 27$. Numbers like 1, 8, 27, 64, 125... are called **perfect cubes** because their cube roots are whole numbers.

21.5 Using Exponents in Scientific Notation (Basic)

Scientific notation is a way to write very large or very small numbers compactly using powers of 10. A number in scientific notation has the form $a \times 10^n$, where a is a number between 1 and 10 (but not 10 itself), and n is an integer.

Step 1: **Identify 'a':** Move the decimal point so there is only one non-zero digit to its left. This new number is 'a'.

Step 2: **Count 'n':** Count how many places you moved the decimal point. This count is 'n'.

- If you moved the decimal to the **left**, n is positive.
- If you moved the decimal to the **right**, n is negative.

Example (Large number): Write 45,000,000 in scientific notation.

- Move decimal left until it's after the 4: 4.5. So $a = 4.5$.
- You moved it 7 places to the left. So $n = 7$.

Result: 4.5×10^7 .

Example (Small number): Write 0.0000000067 in scientific notation.

- Move decimal right until it's after the 6: 6.7. So $a = 6.7$.
- You moved it 9 places to the right. So $n = -9$.

Result: 6.7×10^{-9} .

Why is this important?

Scientific notation is used in science (e.g., distances in space, size of atoms) and engineering.

Practice Problems

1. Evaluate: 3^4
2. Evaluate: 10^0
3. Simplify: $x^5 \times x^3$
4. Simplify: $\frac{y^7}{y^4}$
5. Simplify: $(a^4)^3$
6. Evaluate: 4^{-2}
7. Find the square root: $\sqrt{81}$
8. Find the cube root: $\sqrt[3]{64}$
9. Write in scientific notation: 93,000,000
10. Write in scientific notation: 0.000012

Chapter 22

Introduction to Functions

In math, we often look for relationships between quantities. A "function" is a special kind of relationship that's very common and powerful.

22.1 What Is a Function? (Input/Output, Machine Analogy)

A **function** is a rule that assigns exactly one output for each input. Think of a function as a machine:

- You put something (an **input**) into the machine.
- The machine does something to it (applies the **rule**).
- You get exactly one thing out (an **output**).

We often use $f(x)$ notation (read "f of x") to represent a function, where x is the input. Example: $f(x) = 2x + 1$. If input $x = 3$, output $f(3) = 2(3) + 1 = 6 + 1 = 7$. So, the input 3 gives the output 7.

Why is this important?

The key idea: Each input has ONLY ONE output. If an input gives two different outputs, it's not a function.

22.2 Domain and Range (Basic Understanding)

- **Domain:** The set of all possible **input** values for a function.
- **Range:** The set of all possible **output** values produced by a function.

Example: For a function that maps the number of apples (x) to their total cost (y), the domain might be whole numbers (you can't buy -2 apples). The range would be the possible costs (positive numbers).

22.3 Representing Functions (Tables, Graphs, Equations)

Functions can be shown in several ways:

- **Equation (Rule):** A mathematical formula (e.g., $y = 3x - 5$). This tells you exactly what to do with any input.

- **Table of Values:** A list of input-output pairs. Example for $y = x + 2$:

x (Input)	y (Output)
-1	1
0	2
1	3
2	4

- **Graph:** A visual representation on the coordinate plane, where each point (x, y) represents an input-output pair. (You've already practiced graphing linear equations, which are a type of function.)

22.4 Linear Functions ($y = mx + b$) – Revisiting Graphing Straight Lines)

The general form of a linear function is $y = mx + b$.

- m is the **slope**: It tells you how steep the line is and its direction (upwards or downwards). It's "rise over run."
- b is the **y-intercept**: This is the point where the line crosses the y-axis (where $x = 0$).

Step 1: **Identify the y-intercept (b):** Plot this point on the y-axis.

Step 2: **Use the slope (m) to find other points:** From the y-intercept, use "rise over run" to count units up/down (rise) and right/left (run) to find another point.

Step 3: **Draw the line:** Connect the points with a straight line.

Example: Graph $y = 2x + 1$

- Y-intercept ($b = 1$): Plot $(0, 1)$.
- Slope ($m = 2$, which is $\frac{2}{1}$): From $(0, 1)$, go up 2 units and right 1 unit to find $(1, 3)$.
- Draw a line through $(0, 1)$ and $(1, 3)$.

(This is a faster way to graph linear functions than making a full table, once you understand slope.)

Why is this important?

Functions are fundamental in all higher math and sciences, used to model everything from population growth to planetary motion.

Practice Problems

1. Is the relationship $\{(1, 5), (2, 7), (3, 5)\}$ a function? Why or why not?
2. Is the relationship $\{(1, 5), (1, 7), (3, 9)\}$ a function? Why or why not?
3. For the function $f(x) = 3x - 2$, find $f(5)$ and $f(-1)$.
4. For the linear function $y = -\frac{1}{2}x + 3$:
 - What is the y-intercept?
 - What is the slope?
 - Create a table of values for $x = -2, 0, 2, 4$.
 - Describe how you would graph this function using slope and y-intercept.

Chapter 23

Logic and Problem-Solving Strategies

Math isn't just about crunching numbers; it's about thinking clearly and solving problems. This chapter will introduce you to basic logical reasoning and general strategies that can be applied to any problem, in math or in life.

23.1 Logical Reasoning (If-Then Statements, True/False)

Logic is the study of correct reasoning. It helps us determine if an argument is valid or a statement is true.

- **Statements:** Sentences that are either definitively true or definitively false. Example: "The sky is green." (False statement) Example: " $2 + 2 = 4$." (True statement)
- **If-Then Statements (Conditional Statements):** These link a condition to a consequence. "If P, then Q." P is the hypothesis (the condition). Q is the conclusion (the consequence). Example: "If it rains (P), then the ground gets wet (Q)."

Understanding basic logic helps you break down complex arguments and problem scenarios.

23.2 Problem-Solving Steps (Understand, Plan, Solve, Check)

Many complex problems can be tackled using a systematic approach:

Step 1: Understand the Problem:

- What is the question asking?
- What information is given? What is unknown?
- Are there any keywords or special conditions?
- Can you rephrase the problem in your own words?

Step 2: Plan Your Approach:

- What strategy will you use? (See below)
- What steps do you need to take?

23.3. APPLYING VARIOUS STRATEGIES (DRAW A PICTURE, MAKE A TABLE, LOOK FOR A PATTERN)

- What formulas or concepts might apply?

Step 3: Solve the Problem:

- Carry out your plan.
- Show your work clearly.
- Don't be afraid to try, even if it doesn't work perfectly the first time.

Step 4: Check Your Answer:

- Does your answer make sense in the context of the problem?
- Is it reasonable? (e.g., if you're finding the number of apples, a negative number isn't reasonable).
- Did you answer the original question?
- Can you check your work using a different method?

23.3 Applying Various Strategies (Draw a Picture, Make a Table, Look for a Pattern, Work Backward, Guess and Check)

Here are some common problem-solving strategies:

- **Draw a Picture/Diagram:** Visualizing the problem can often reveal hidden relationships.
- **Make a Table/List:** Organizing information systematically can help you see patterns or track possibilities.
- **Look for a Pattern:** Many problems involve sequences or repeating elements.
- **Work Backward:** Start from the end result and undo each step to find the beginning.
- **Guess and Check (and Revise):** Make an educated guess, test it, and then use the results to make a better guess. **Simplify the Problem:** Break a big problem into smaller, more manageable parts.
- **Use an Equation/Formula:** Translate the problem into an algebraic equation.

23.4 Advanced Word Problems Integrating Concepts from Previous Chapters.

This is where all your basic math skills come together!

Real-World Example:

"A rectangular garden has a perimeter of 40 feet. The length is 4 feet more than twice the width. Find the dimensions (length and width) of the garden."

- ****Understand:**** Need length and width. Given perimeter and relationship between length and width.
- ****Plan:**** Use variables for length and width. Set up equations using perimeter formula and the given relationship. Solve the system of equations.
- ****Solve:**** Let W = width. Length $L = 2W + 4$. Perimeter $P = 2L + 2W$. Substitute L into the perimeter formula: $40 = 2(2W + 4) + 2W$. $40 = 4W + 8 + 2W$ (Distributive Property) $40 = 6W + 8$ (Combine like terms) $40 - 8 = 6W$ $32 = 6W$ $W = \frac{32}{6} = \frac{16}{3} = 5\frac{1}{3}$ feet. Now find L : $L = 2(\frac{16}{3}) + 4 = \frac{32}{3} + \frac{12}{3} = \frac{44}{3} = 14\frac{2}{3}$ feet.
- ****Check:**** Perimeter $= 2(14\frac{2}{3}) + 2(5\frac{1}{3}) = 2(\frac{44}{3}) + 2(\frac{16}{3}) = \frac{88}{3} + \frac{32}{3} = \frac{120}{3} = 40$. (Correct!)
- ****State:**** The width of the garden is $5\frac{1}{3}$ feet and the length is $14\frac{2}{3}$ feet.

Practice Problems

1. Is the statement "If a shape is a square, then it has four equal sides" true or false?
2. Use the problem-solving steps to solve: "You have a bag of marbles. If you double the number of marbles, then subtract 7, you get 21 marbles. How many marbles were in the bag initially?"
3. Describe how you would use the "draw a picture" strategy to solve a problem about finding the shortest path between two points in a maze.
4. A store is having a 20% off sale. You buy a shirt for \$25. The sales tax is 5%. What is the final price you pay for the shirt? (Combine percentages, multiplication, and addition).

Chapter 24

Financial Literacy Basics

Math is incredibly important for managing your money. Understanding basic financial concepts helps you make smart decisions throughout your life.

24.1 Budgeting and Saving

- **Budgeting:** Creating a plan for how you will spend and save your money. It involves tracking your income (money coming in) and expenses (money going out).

Step 1: Calculate Income: Total money you expect to receive.

Step 2: List Expenses: All your regular payments (rent, food, phone, entertainment, etc.).

Step 3: Categorize & Track: Group expenses and regularly track where your money goes.

Step 4: Compare Income & Expenses: Ideally, income > expenses. If not, adjust expenses.

- **Saving:** Setting aside money for future goals.
 - **Emergency Fund:** Money saved for unexpected costs.
 - **Short-Term Goals:** For things like a new phone, concert tickets.
 - **Long-Term Goals:** For big purchases like a car, house, or retirement.

Real-World Example:

If your monthly income is \$1,000 and your expenses are \$800, you have \$200 left over to save or spend.

24.2 Simple Interest Calculation

Interest is the cost of borrowing money or the earnings from saving money. **Simple interest** is calculated only on the original amount (the principal). Formula: $I = P \times R \times T$

- I = Interest earned/paid
- P = Principal (the initial amount of money)

- R = Annual Interest Rate (as a decimal)
- T = Time (in years)

Real-World Example:

You deposit \$1,000 into a savings account with a simple annual interest rate of 3%. How much interest will you earn in 2 years? $P = \$1,000$ $R = 3\% = 0.03$ (as a decimal) $T = 2$ years $I = \$1,000 \times 0.03 \times 2 = \$30 \times 2 = \$60$. You will earn \$60 in interest.

24.3 Understanding Percentages in Personal Finance (Discounts, Tax, Interest)

Percentages are vital for financial math:

- **Discounts/Sales:** "30% off" means you subtract 30% of the original price.
- **Sales Tax:** A percentage added to the price of goods. Calculate the tax amount, then add it to the price.
- **Tips:** A percentage of the bill (often 15-20%) added for service.
- **Interest Rates:** Expressed as percentages (e.g., 5% interest). Remember to convert them to decimals for calculations.

Real-World Example:

A sweater costs \$45. There's a 10% discount and then a 6% sales tax.

- Discount amount: $0.10 \times \$45 = \4.50 .
- Price after discount: $\$45 - \$4.50 = \$40.50$.
- Sales tax amount: $0.06 \times \$40.50 = \2.43 .
- Final price: $\$40.50 + \$2.43 = \$42.93$.

24.4 Introduction to Credit (Basic Concepts)

Credit is the ability to borrow money or buy goods/services with a promise to pay later.

- **Loan:** Borrowing a lump sum of money that you pay back over time, usually with interest.
- **Credit Card:** A card that allows you to borrow money up to a certain limit. If you don't pay the full balance each month, you're charged high interest.
- **Credit Score:** A number that indicates how risky you are as a borrower. A good credit score is important for loans, housing, and even some jobs.

Why is this important?

Understanding financial math allows you to control your money, rather than letting your money control you!

Practice Problems

1. Your monthly income is \$1,500. Your rent is \$600, food is \$300, transportation is \$150, and entertainment is \$100. How much money do you have left for savings each month?
2. You invest \$5,000 in a certificate of deposit (CD) that offers a simple annual interest rate of 2.5%. How much interest will you earn after 3 years?
3. A pair of shoes costs \$80. They are on sale for 25% off. What is the sale price?
4. If the sales tax is 8%, how much tax would you pay on an item that costs \$50? What is the total cost?
5. Explain in your own words why having a good credit score is important.

Appendix

Glossary of Math Terms

- **Absolute Value:** The distance of a number from zero on the number line, always non-negative.
- **Acute Angle:** An angle measuring less than 90° .
- **Addition:** The process of combining two or more numbers to find their sum.
- **Algebra:** A branch of mathematics using letters (variables) to represent numbers and solve problems. **Algebraic Expression:** A mathematical phrase that contains numbers, variables, and operation symbols, but no equality or inequality sign.
- **Angle:** Formed by two rays sharing a common endpoint (vertex).
- **Area:** The amount of surface inside a 2D shape.
- **Average (Mean):** The sum of all values in a data set divided by the number of values.
- **Bar Graph:** A graph using bars to show the frequency of different categories.
- **Base (exponent):** The number being multiplied repeatedly in an exponential expression.
- **Budgeting:** Creating a plan for managing income and expenses.
- **Capacity:** The amount a container can hold (liquid volume).
- **Celsius:** A metric unit of temperature.
- **Center (of a circle):** The central point from which all points on the circle are equidistant.
- **Centi-:** A metric prefix meaning one hundredth (0.01).
- **Certain Event:** An event that will definitely happen. Probability = 1.
- **Circle:** A set of all points in a plane equidistant from a central point.
- **Circle Graph (Pie Chart):** A graph divided into sectors representing proportions of a whole.
- **Circumference:** The distance around a circle.

- **Coefficient:** The numerical factor of a term that contains a variable.
- **Common Denominator:** A common multiple of the denominators of two or more fractions.
- **Cone:** A 3D shape with a circular base tapering to a point.
- **Constant:** A fixed value that does not change.
- **Coordinate Plane:** A two-dimensional plane defined by two perpendicular number lines (axes).
- **Credit:** The ability to borrow money or buy goods/services with a promise to pay later.
- **Credit Card:** A card allowing borrowing up to a limit, with interest charged if not fully paid.
- **Credit Score:** A number indicating a borrower's creditworthiness.
- **Cross-Multiplication:** A method used to solve proportions: $a/b = c/d \implies ad = bc$.
- **Cube (3D):** A 3D shape with 6 square faces.
- **Cube Root:** A value that, when multiplied by itself three times, gives the original number.
- **Customary System:** A system of measurement primarily used in the U.S., with units like inches, pounds, gallons.
- **Data:** Information, often numerical, collected for analysis.
- **Decimal:** A number that uses a decimal point to show parts of a whole (tenths, hundredths, etc.). **Denominator:** The bottom number in a fraction, indicating how many equal parts the whole is divided into.
- **Difference:** The result of a subtraction problem.
- **Diameter:** The distance across a circle through its center ($d = 2r$).
- **Discount:** A reduction in price, usually a percentage off.
- **Distributive Property:** A rule stating that $a(b + c) = ab + ac$.
- **Division:** The process of splitting a number into equal parts or groups.
- **Domain:** The set of all possible input values for a function.
- **Equally Likely Event:** An event with a 50% chance of happening. Probability = 0.5.
- **Equation:** A mathematical statement that shows two expressions are equal.
- **Equivalent Fractions:** Different fractions that represent the same value.

- **Equivalent Ratios:** Ratios that represent the same relationship.
- **Even Number:** A whole number that can be divided exactly by 2 (ends in 0, 2, 4, 6, or 8).
- **Event (probability):** A set of one or more outcomes.
- **Expenses:** Money spent on various items or services.
- **Exponent:** A small number written above and to the right of a base number, indicating how many times the base number is multiplied by itself.
- **Fahrenheit:** A customary unit of temperature.
- **Financial Literacy:** The ability to understand and effectively use various financial skills.
- **Fraction:** A way to represent a part of a whole or a part of a set, written as a numerator over a denominator.
- **Frequency Table:** A table listing categories of data and their counts.
- **Function:** A rule that assigns exactly one output for each input.
- **Glossary:** A list of terms with their definitions.
- **Greatest Common Factor (GCF):** The largest number that divides evenly into two or more numbers.
- **Hexagon:** A polygon with 6 sides.
- **Hypotenuse:** The longest side of a right triangle, opposite the right angle.
- **Impossible Event:** An event that will never happen. Probability = 0.
- **Improper Fraction:** A fraction where the numerator is equal to or larger than the denominator.
- **Income:** Money received, especially on a regular basis. **Inequality:** A mathematical statement comparing expressions using symbols like $<$, $>$, $\leq$, $\geq$.
- **Input:** A value that goes into a function.
- **Integer:** A whole number that can be positive, negative, or zero (e.g., -3, 0, 5).
- **Interest:** The cost of borrowing money or the earnings from saving money.
- **Intersecting Lines:** Lines that cross each other at exactly one point.
- **Inverse Operation:** An operation that undoes another (e.g., addition undoes subtraction).
- **Kilo-:** A metric prefix meaning one thousand (1,000) times the base unit.
- **Least Common Denominator (LCD):** The least common multiple of the denominators of two or more fractions.

- **Legs (of a right triangle):** The two shorter sides that form the right angle.
- **Length:** The measurement of how long something is.
- **Likely Event:** An event that has a high chance of happening. Probability is close to 1. **Like Terms:** Terms that have the exact same variable parts.
- **Line:** A perfectly straight path extending infinitely in both directions.
- **Line Graph:** A graph using points connected by lines to show data change over time.
- **Line Segment:** A part of a line with two endpoints.
- **Line Symmetry:** Property of a shape that can be folded along a line to create matching halves.
- **Linear Equation:** An equation whose graph is a straight line.
- **Loan:** Borrowed money that must be repaid, usually with interest.
- **Logic:** The study of correct reasoning.
- **Mass:** A measure of the amount of matter in an object (related to weight).
- **Mean:** The average of a set of numbers, found by summing all numbers and dividing by the count of numbers.
- **Measurement:** The process of quantifying physical properties.
- **Median:** The middle value in an ordered set of numbers.
- **Metric System:** A system of measurement based on powers of 10, with units like meters, liters, grams. **Milli-:** A metric prefix meaning one thousandth (0.001).
- **Mixed Number:** A combination of a whole number and a proper fraction.
- **Mode:** The value that appears most frequently in a set of data.
- **Multiplication:** The process of combining equal groups, often seen as repeated addition.
- **Negative Number:** A number less than zero.
- **Number Line:** A visual representation of numbers as points on a straight line.
Numerator: The top number in a fraction, telling how many parts are being considered.
- **Obtuse Angle:** An angle measuring greater than 90° but less than 180° .
- **Octagon:** A polygon with 8 sides.
- **Odd Number:** A whole number that cannot be divided exactly by 2 (ends in 1, 3, 5, 7, or 9).

- **Order of Operations:** The specific sequence in which mathematical operations must be performed (PEMDAS/BODMAS).
- **Ordered Pair:** A pair of numbers (x, y) used to locate a point on a coordinate plane.
- **Origin:** The point (0,0) where the x-axis and y-axis intersect.
- **Outcome (probability):** A single possible result of an experiment.
- **Output:** The value produced by a function for a given input.
- **Parallel Lines:** Lines that are always the same distance apart and never intersect.
- **Parallelogram:** A quadrilateral with two pairs of parallel sides.
- **Pentagon:** A polygon with 5 sides. **Percentage:** A fraction out of 100, represented by the % symbol.
- **Perimeter:** The total distance around the outside edge of a 2D shape.
- **Perpendicular Lines:** Lines that intersect to form right angles.
- **Pi (π):** The ratio of a circle's circumference to its diameter, approximately 3.14159.
- **Pictograph:** A graph using pictures or symbols to represent data.
- **Place Value:** The value of a digit based on its position in a number.
- **Point:** An exact location in space, having no size.
- **Polygon:** A closed 2D shape made of straight line segments.
- **Positive Number:** A number greater than zero.
- **Problem-Solving Strategies:** Systematic approaches to finding solutions.
- **Probability:** A measure of how likely an event is to occur.
- **Product:** The result of a multiplication problem.
- **Proper Fraction:** A fraction where the numerator is smaller than the denominator. **Proportion:** An equation stating that two ratios are equal.
- **Pyramid:** A 3D shape with a polygon base and triangular faces meeting at an apex.
- **Pythagorean Theorem:** In a right triangle, $a^2 + b^2 = c^2$.
- **Quadrilateral:** A polygon with 4 sides.
- **Quantity:** An amount or number of something.
- **Quotient:** The result of a division problem.
- **Radius:** The distance from the center of a circle to any point on its circumference.

- **Range (data):** The difference between the highest and lowest values in a data set. **Rate:** A ratio comparing two quantities with different units. **Ratio:** A comparison of two or more quantities.
- **Ray:** Part of a line with one endpoint, extending infinitely in one direction.
- **Rectangle:** A quadrilateral with four right angles.
- **Rectangular Prism:** A 3D shape with 6 rectangular faces.
- **Reflection (Flip):** A transformation that creates a mirror image of a shape.
- **Reflex Angle:** An angle measuring greater than 180° but less than 360° .
- **Remainder:** The amount left over after a division when the numbers don't divide evenly.
- **Right Angle:** An angle measuring exactly 90° .
- **Right Triangle:** A triangle with one right angle.
- **Rotation (Turn):** A transformation that turns a shape around a fixed point.
- **Rotational Symmetry:** Property of a shape that looks the same after a partial turn. **Rounding:** Approximating a number to a simpler value.
- **Sales Tax:** A percentage added to the price of goods.
- **Saving:** Setting aside money for future goals. **Scientific Notation:** A way to write very large or small numbers using powers of 10.
- **Simplifying Fraction:** Reducing a fraction to its lowest terms.
- **Simple Interest:** Interest calculated only on the original principal amount.
- **Slope:** A measure of the steepness and direction of a line on a graph. **Solving an Equation:** Finding the value(s) of the variable(s) that make the equation true.
- **Sphere:** A perfectly round 3D object.
- **Square Root:** A value that, when multiplied by itself, gives the original number.
- **Straight Angle:** An angle measuring exactly 180° .
- **Subtraction:** The process of taking one number away from another to find the difference.
- **Sum:** The result of an addition problem.
- **Tally Chart:** A chart using marks to count occurrences of data.
- **Temperature:** A measure of hotness or coldness. **Term (algebra):** A single number, variable, or product in an algebraic expression.
- **Time:** A measure of duration. **Tip:** A percentage of a bill given for service.

- **Transformation:** A way to move a geometric shape without changing its size or shape. **Translation (Slide):** A transformation that moves a shape without rotation or flipping.
- **Triangle:** A polygon with 3 sides.
- **Unlikely Event:** An event that has a low chance of happening. Probability is close to 0. **Unit Rate:** A rate where the second quantity is 1 unit. **Variable:** A letter or symbol used to represent an unknown quantity in an algebraic expression or equation. **Vertex:** The common endpoint of two rays forming an angle, or a corner of a polygon.
- **Volume:** The amount of space a 3D object occupies.
- **Weight:** A measure of how heavy something is.
- **x-axis:** The horizontal number line on a coordinate plane.
- **y-axis:** The vertical number line on a coordinate plane. **Y-intercept:** The point where a line crosses the y-axis.

Formulas Quick Reference

- **Area of a Rectangle:** $A = \text{length} \times \text{width}$ ($A = l \times w$)
- **Perimeter of a Rectangle:** $P = 2 \times (\text{length} + \text{width})$ ($P = 2(l + w)$)
- **Area of a Square:** $A = \text{side}^2$ ($A = s^2$)
- **Perimeter of a Square:** $P = 4 \times \text{side}$ ($P = 4s$)
- **Area of a Triangle:** $A = \frac{1}{2} \times \text{base} \times \text{height}$ ($A = \frac{1}{2}bh$)
- **Perimeter of a Triangle:** $P = \text{side}_1 + \text{side}_2 + \text{side}_3$
- **Area of a Parallelogram:** $A = \text{base} \times \text{height}$ ($A = bh$)
- **Circumference of a Circle:** $C = \pi \times \text{diameter}$ ($C = \pi d$) or $C = 2 \times \pi \times \text{radius}$ ($C = 2\pi r$)
- **Area of a Circle:** $A = \pi \times \text{radius}^2$ ($A = \pi r^2$)
- **Volume of a Rectangular Prism:** $V = \text{length} \times \text{width} \times \text{height}$ ($V = lwh$)
- **Volume of a Cube:** $V = \text{side}^3$ ($V = s^3$)
- **Pythagorean Theorem (for right triangles):** $a^2 + b^2 = c^2$
- **Simple Probability:** $P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$ **Mean (Average):**
 $\text{Mean} = \frac{\text{Sum of all values}}{\text{Number of values}}$
- **Simple Interest:** $I = PRT$ (Interest = Principal $\times$ Rate $\times$ Time) **Linear Function (Slope-Intercept Form):** $y = mx + b$

Practice Problem Solutions

Solutions for Chapter 1: What Are Numbers?

Solutions

1. The '7' in 472 is in the tens place, so its value is 70.
2. 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15.
3. (Visual representation of number line with 6 marked)
4. $38 < 83$
5. $101 > 99$
6. $50 + 5 = 55$
7. 47 is an **odd** number (ends in 7). 120 is an **even** number (ends in 0).

Solutions for Chapter 2: All About Addition (+) and Subtraction (-)

Solutions

1. $15 + 7 = 22$
2. $28 - 9 = 19$
3. Add: $146 + 32 = 178$
4. Subtract: $58 - 25 = 33$
5. Add: $39 + 48 = 87$
6. Subtract: $73 - 18 = 55$
7. Maria has $\$25 + \$15 = \$40$.
8. There are $30 - 12 = 18$ birds left.

Solutions for Chapter 3: Introduction to Multiplication (*)

Solutions

1. $5 \times 3 = 3 + 3 + 3 + 3 + 3 = 15$
2. $7 \times 8 = 56$
3. $25 \times 100 = 2500$
4. $53 \times 4 = 212$
5. $13 \times 21 = 273$
6. A classroom has 6 tables. Each table has 4 chairs. How many chairs are there in total?
 $6 \times 4 = 24$ chairs.

Solutions for Chapter 4: Understanding Division (/)

Solutions

1. $48 \div 6 = 8$
2. $25 \div 4 = 6 \text{ R } 1$
3. $96 \div 8 = 12$
4. $435 \div 15 = 29$
5. A baker made 72 cookies and wants to put them into bags, with 9 cookies per bag. How many bags does he need? $72 \div 9 = 8$ bags.
6. A car travels 60 miles per hour. How far does it travel in 3 hours? $60 \text{ miles/hour} \times 3 \text{ hours} = 180 \text{ miles}$.

Solutions for Chapter 5: Order of Operations (PEMDAS/BODMAS)

Solutions

1. $5 + 2 \times 3 = 5 + 6 = 11$

2. $(10 - 2) \div 4 = 8 \div 4 = 2$

3. $20 - 6 \div 2 + 5 = 20 - 3 + 5 = 17 + 5 = 22$

4. $4 \times (3 + 1) - 10 \div 5 = 4 \times 4 - 10 \div 5 = 16 - 2 = 14$

5. $3^2 + (8 - 4) \times 2 = 9 + 4 \times 2 = 9 + 8 = 17$

Solutions for Chapter 6: Deep Dive into Fractions

Solutions

1. $\frac{3}{5}$ (3 stars out of 5 total shapes)
2. Numerator = 5, Denominator = 9
3. $17 \div 5 = 3$ with remainder 2, so $3\frac{2}{5}$.
4. $(3 \times 4) + 1 = 13$, so $\frac{13}{4}$.
5. $\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$
6. Yes. $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$.
7. $\frac{1}{4} = \frac{3}{12}$, $\frac{2}{3} = \frac{8}{12}$. So $\frac{3}{12} < \frac{8}{12}$, which means $\frac{1}{4} < \frac{2}{3}$.
8. $\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$
9. LCD is 10. $\frac{1}{5} = \frac{2}{10}$. So $\frac{7}{10} - \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$.
10. $\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$
11. $\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{15}{12} = \frac{5}{4}$ or $1\frac{1}{4}$.

Solutions for Chapter 7: Working with Decimals

Solutions

1. Hundredths place.
2. $1 \div 4 = 0.25$.
3. 0.75 is 75 hundredths, so $\frac{75}{100}$. Simplified: $\frac{75 \div 25}{100 \div 25} = \frac{3}{4}$.
4. 0.09, 0.1, 0.11 (Think of them as 9, 10, 11 if all were thousandths: 0.100, 0.090, 0.110).
5. $12.30 + 4.75 = 17.05$.
6. $5.0 - 2.8 = 2.2$.
7. $0.7 \times 0.4 = 0.28$ ($7 \times 4 = 28$, 2 decimal places).
8. $25.5 \div 5 = 5.1$.
9. $6.2 \times 100 = 620$ (Move decimal 2 places right).
10. $14.7 \div 10 = 1.47$ (Move decimal 1 place left).
11. Round 7.863 to nearest tenth: The tenth is '8'. The digit to its right is '6' (5 or greater), so round up the '8' to '9'. Result: 7.9.

Solutions for Chapter 8: Exploring Integers

Solutions

1. $-7 < -2$ (because -7 is further left on the number line)

2. -10

3. $10 + (-4) = 6$

4. $(-6) + (-8) = -14$

5. $7 - 12 = 7 + (-12) = -5$

6. $-3 - (-9) = -3 + 9 = 6$

7. $5 \times (-6) = -30$ (different signs, so negative)

8. $(-20) \div (-4) = 5$ (same signs, so positive)

9. $|-15| = 15$

10. $|15| - |-5| = 15 - 5 = 10$

Solutions for Chapter 9: Ratios, Rates, and Proportions

Solutions

1. $4 : 6$, 4 to 6, or $\frac{4}{6}$ (which simplifies to $2 : 3$, 2 to 3, or $\frac{2}{3}$)
2. $1 : 3$ (divide by 4); $8 : 24$ (multiply by 2). Many other answers are possible.
3. $\frac{200 \text{ miles}}{4 \text{ hours}} = 50 \text{ miles/hour}$.
4. $\frac{\$1.50}{3 \text{ pencils}} = \0.50 per pencil .
5. $\frac{x}{5} = \frac{9}{15} \implies 15x = 5 \times 9 \implies 15x = 45 \implies x = \frac{45}{15} = 3$.
6. A recipe calls for 2 cups of sugar for every 3 cups of flour. If you use 6 cups of flour, how much sugar do you need? $\frac{2 \text{ cups sugar}}{3 \text{ cups flour}} = \frac{x \text{ cups sugar}}{6 \text{ cups flour}} \implies 3x = 2 \times 6 \implies 3x = 12 \implies x = 4$. You need 4 cups of sugar.

Solutions for Chapter 10: Understanding Percentages (%)

Solutions

1. Decimal: 0.45. Fraction: $\frac{45}{100} = \frac{9}{20}$.
2. Percentage: 8%. Fraction: $\frac{8}{100} = \frac{2}{25}$.
3. Decimal: 0.2. Percentage: 20%.
4. What is 15% of 200? $0.15 \times 200 = 30$.
5. 24 is 40% of what number? $24 \div 0.40 = 60$.
6. A shirt originally cost \$40. It is now on sale for \$30. What is the percentage decrease?
Decrease = $\$40 - \$30 = \$10$. Percentage Decrease = $\frac{\$10}{\$40} = 0.25 = 25\%$.
7. You want to leave a 20% tip on a \$55 bill. How much is the tip? Tip = $0.20 \times \$55 = \11 .

Solutions for Chapter 11: Measurement and Units

Solutions

1. Customary: inches, Metric: centimeters.
2. How many centimeters are in 3 meters? 1 meter = 100 cm. So, $3 \text{ meters} \times 100 \text{ cm/meter} = 300 \text{ cm}$.
3. How many ounces are in 2 pounds? 1 pound = 16 ounces. So, $2 \text{ pounds} \times 16 \text{ ounces/pound} = 32 \text{ ounces}$.
4. Convert 4.5 liters to milliliters. 1 liter = 1000 milliliters. So, $4.5 \text{ liters} \times 1000 \text{ ml/liter} = 4500 \text{ ml}$.
5. Convert 36 inches to yards. 1 yard = 36 inches. So, $36 \text{ inches} \div 36 \text{ inches/yard} = 1 \text{ yard}$.
6. If 1 kilogram is approximately 2.2 pounds, how many pounds is 5 kilograms? $5 \text{ kilograms} \times 2.2 \text{ pounds/kilogram} = 11 \text{ pounds}$.
7. A recipe calls for 1.5 cups of milk. How many fluid ounces is that? 1 cup = 8 fluid ounces. So, $1.5 \text{ cups} \times 8 \text{ fluid ounces/cup} = 12 \text{ fluid ounces}$.

Solutions for Chapter 12: Algebraic Expressions

Solutions

1. In the expression $7a - 3b + 12$:

- Variables: a, b .
- Constants: 12.
- Coefficients: 7, -3 .
- Terms: 3 (these are $7a$, $-3b$, and 12).

2. Translate into an algebraic expression:

- "A number decreased by 9": $x - 9$.
- "The product of 5 and a number": $5x$.
- "7 more than half a number": $\frac{x}{2} + 7$ or $0.5x + 7$.

3. Evaluate the expression $2x^2 + 5x - 1$ when $x = 3$. $2(3)^2 + 5(3) - 1 = 2(9) + 15 - 1 = 18 + 15 - 1 = 33 - 1 = 32$.

4. Simplify by combining like terms: $8p + 3q - 5p + 2q$ $(8p - 5p) + (3q + 2q) = 3p + 5q$.

5. Simplify by combining like terms: $10 - 4m + 7 + 2m$ $(10 + 7) + (-4m + 2m) = 17 - 2m$.

6. Apply the distributive property: $6(y + 4)$ $6 \times y + 6 \times 4 = 6y + 24$.

7. Apply the distributive property: $3(2x - 7)$ $3 \times 2x - 3 \times 7 = 6x - 21$.

Solutions for Chapter 13: Solving Basic Equations

Solutions

1. Solve for x : $x + 10 = 25 \implies x = 25 - 10 \implies x = 15$.
2. Solve for y : $y - 7 = 15 \implies y = 15 + 7 \implies y = 22$.
3. Solve for a : $6a = 42 \implies a = \frac{42}{6} \implies a = 7$.
4. Solve for b : $\frac{b}{5} = 10 \implies b = 10 \times 5 \implies b = 50$.
5. Solve for m : $3m - 4 = 17 \implies 3m = 17 + 4 \implies 3m = 21 \implies m = \frac{21}{3} \implies m = 7$.
6. Solve for n : $\frac{n}{2} + 5 = 9 \implies \frac{n}{2} = 9 - 5 \implies \frac{n}{2} = 4 \implies n = 4 \times 2 \implies n = 8$.
7. Solve for p : $7p - 1 = 4p + 11 \implies 7p - 4p = 11 + 1 \implies 3p = 12 \implies p = \frac{12}{3} \implies p = 4$.
8. Twice a number, increased by 5, is 19. What is the number? (Set up and solve an equation) Let the number be x . Equation: $2x + 5 = 19$. $2x = 19 - 5 \implies 2x = 14 \implies x = 7$. The number is 7.
9. A rectangle has a perimeter of 30 cm. Its length is 10 cm. What is its width? (Hint: Perimeter $= 2L + 2W$) Let width be W . Perimeter $P = 2L + 2W$. Given $P = 30$, $L = 10$. $30 = 2(10) + 2W \implies 30 = 20 + 2W \implies 10 = 2W \implies W = 5$. The width is 5 cm.

Solutions for Chapter 14: Inequalities and Graphing

Solutions

1. Write the inequality for "You need to be at most 16 years old to get a junior driver's license." Let age be A . $A \leq 16$.
2. Graph $x > 4$ on a number line. (Open circle at 4, arrow to the right).
3. Graph $x \leq -2$ on a number line. (Closed circle at -2, arrow to the left).
4. Solve $x + 3 < 7 \implies x < 4$. Graph: (Open circle at 4, arrow to the left).
5. Solve $5y \geq 15 \implies y \geq 3$. Graph: (Closed circle at 3, arrow to the right).
6. Solve $-2m < 10 \implies m > -5$ (Divided by a negative, so reversed sign). Graph: (Open circle at -5, arrow to the right).
7. Plot the following points on a coordinate plane: A(4, 1), B(-2, 3), C(0, -4), D(-1, -1). (Description of points on a 2D plane). A in Quadrant I, B in Quadrant II, C on y-axis, D in Quadrant III.
8. Create a table of values and graph the linear equation $y = 2x - 1$. Table for $y = 2x - 1$:

x	$y = 2x - 1$
0	$2(0) - 1 = -1$
1	$2(1) - 1 = 1$
2	$2(2) - 1 = 3$
-1	$2(-1) - 1 = -3$

(Then describe drawing a line through these points.)

Solutions for Chapter 15: Fundamental Geometric Concepts

Solutions

1. (Drawings would be needed here. For example: X–Y for line segment; an arrow extending from B through A for ray AB)
2. What type of angle is 45° ? What type is 135° ? 45° is an **acute** angle. 135° is an **obtuse** angle.
3. If two lines never meet, what are they called? Parallel lines.
4. How many sides does a hexagon have? A hexagon has 6 sides.
5. If a circle has a radius of 5 cm, what is its diameter? Diameter = $2 \times \text{radius} = 2 \times 5 \text{ cm} = 10 \text{ cm}$.
6. If a circle has a diameter of 10 meters, what is its circumference (use $\pi \approx 3.14$)? Circumference = $\pi \times \text{diameter} = 3.14 \times 10 \text{ m} = 31.4 \text{ m}$.

Solutions for Chapter 16: Perimeter, Area, and Volume

Solutions

1. A square garden has sides of 12 meters. What is its perimeter and area? Perimeter $P = 4s = 4 \times 12 \text{ m} = 48 \text{ m}$. Area $A = s^2 = 12^2 \text{ m}^2 = 144 \text{ m}^2$.
2. A rectangular swimming pool is 25 feet long and 10 feet wide. What is the area of the water surface? Area $A = L \times W = 25 \text{ ft} \times 10 \text{ ft} = 250 \text{ ft}^2$.
3. Find the perimeter of a triangle with sides 8 cm, 15 cm, and 17 cm. Perimeter $P = 8 + 15 + 17 = 40 \text{ cm}$.
4. Calculate the area of a triangle with a base of 6 inches and a height of 8 inches. Area $A = \frac{1}{2}bh = \frac{1}{2} \times 6 \text{ in} \times 8 \text{ in} = 24 \text{ in}^2$.
5. A circular mat has a radius of 7 cm. What is its area? (Use $\pi \approx \frac{22}{7}$) Area $A = \pi r^2 = \frac{22}{7} \times (7 \text{ cm})^2 = \frac{22}{7} \times 49 \text{ cm}^2 = 22 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$.
6. How many cubic meters of concrete are needed to fill a rectangular foundation that is 10 m long, 5 m wide, and 2 m deep? Volume $V = L \times W \times H = 10 \text{ m} \times 5 \text{ m} \times 2 \text{ m} = 100 \text{ m}^3$.

Solutions for Chapter 17: Symmetry, Transformations, and Basic Pythagorean Theorem

Solutions

1. Does the letter 'E' have line symmetry? If so, how many lines of symmetry? Yes, the letter 'E' has one line of symmetry (horizontal).
2. Does a regular pentagon have rotational symmetry? If so, what is its order? Yes, a regular pentagon has rotational symmetry of order 5 (it looks the same after rotations of 72° , 144° , 216° , 288°).
3. Describe the transformation if a shape is moved straight up 3 units and 2 units to the left. This is a **translation** (slide).
4. A triangle has sides of length 3, 4, and 5. Is it a right triangle? Show your work. Check using Pythagorean Theorem: $a^2 + b^2 = c^2$. Is $3^2 + 4^2 = 5^2$? $9 + 16 = 25$ $25 = 25$. Yes, it is a right triangle.
5. A right triangle has legs measuring 5 cm and 12 cm. What is the length of its hypotenuse? $a^2 + b^2 = c^2 \implies 5^2 + 12^2 = c^2 \implies 25 + 144 = c^2 \implies 169 = c^2 \implies c = \sqrt{169} = 13$ cm. The hypotenuse is 13 cm.
6. If the hypotenuse of a right triangle is 13 inches and one leg is 5 inches, what is the length of the other leg? $a^2 + b^2 = c^2 \implies 5^2 + b^2 = 13^2 \implies 25 + b^2 = 169 \implies b^2 = 169 - 25 \implies b^2 = 144 \implies b = \sqrt{144} = 12$ inches. The other leg is 12 inches.

Solutions for Chapter 18: Collecting and Representing Data

Solutions

1. You are collecting data on the colors of cars in a parking lot. How would you record this data using a tally chart? (List 3 colors with example tallies). Example Tally

Chart (your tallies may vary):

Color	Tally	Frequency
Red	I	6
Blue	II	7
White	III	8

2. Create a simple bar graph from the following data: (Apples: 5, Oranges: 3, Bananas: 7) Bar Graph Description:

- Title: Favorite Fruits
- X-axis: Fruit Type (Apples, Oranges, Bananas)
- Y-axis: Number of People (scale from 0 to 8, marked by 1s)
- Bars: Apple bar goes up to 5, Orange bar to 3, Banana bar to 7.

3. What type of graph would be best to show how a student's height has changed over the past 5 years? Why? A **line graph** would be best because it shows change (trend) over time.
4. A pie chart shows that 30% of students walk to school, 50% ride the bus, and 20% are driven. If there are 200 students, how many walk to school? Students who walk = 30% of 200 = $0.30 \times 200 = 60$ students.

Solutions for Chapter 19: Analyzing Data – Mean, Median, Mode, Range

Solutions

1. Find the mean, median, mode, and range for the following set of numbers representing daily temperatures (in degrees Celsius): 18, 22, 20, 25, 18, 23. Data: 18, 22, 20, 25, 18, 23
 - **Mean:** $\text{Sum} = 18 + 22 + 20 + 25 + 18 + 23 = 126$. $\text{Count} = 6$. $\text{Mean} = \frac{126}{6} = 21$.
 - **Median:** Ordered data: 18, 18, **20, 22**, 23, 25. $\text{Median} = \frac{20+22}{2} = \frac{42}{2} = 21$.
 - **Mode:** 18 appears twice. $\text{Mode} = 18$.
 - **Range:** $\text{Highest} = 25$, $\text{Lowest} = 18$. $\text{Range} = 25 - 18 = 7$.
2. A student's quiz scores are: 7, 8, 9, 10, 6, 8, 7, 9, 8, 10. Find the mean, median, mode, and range. Ordered: 6, 7, 7, 8, 8, 8, 9, 9, 10, 10
 - **Mean:** $\text{Sum} = 6 + 7 + 7 + 8 + 8 + 8 + 9 + 9 + 10 + 10 = 82$. $\text{Count} = 10$. $\text{Mean} = \frac{82}{10} = 8.2$.
 - **Median:** Middle two numbers are 8 and 8. $\text{Median} = \frac{8+8}{2} = 8$.
 - **Mode:** 8 appears three times. $\text{Mode} = 8$.
 - **Range:** $\text{Highest} = 10$, $\text{Lowest} = 6$. $\text{Range} = 10 - 6 = 4$.
3. Why might the median salary be a better representation of "average" salary than the mean in a company where the CEO earns significantly more than everyone else? The median salary would be better because the CEO's very high salary (an outlier) would pull the mean salary up significantly, making it seem like the "average" employee earns more than they actually do. The median is less affected by extreme values.

Solutions for Chapter 20: Introduction to Probability

Solutions

1. A spinner has 4 equal sections: Red, Blue, Green, Yellow.
 - What is the probability of landing on Red? $P(\text{Red}) = \frac{1 \text{ (Red section)}}{4 \text{ (total sections)}} = \frac{1}{4}$.
 - What is the probability of landing on a primary color (Red, Blue, Yellow)? Primary colors: Red, Blue, Yellow (3 sections). $P(\text{Primary Color}) = \frac{3}{4}$.
 - Describe the probability of landing on Orange using words (impossible, likely, etc.). Landing on Orange is **impossible**.
2. You have a deck of 52 playing cards.
 - What is the probability of drawing an Ace? There are 4 Aces (Ace of Spades, Clubs, Hearts, Diamonds). $P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$.
 - What is the probability of drawing a red card? There are 26 red cards (13 Hearts, 13 Diamonds). $P(\text{Red Card}) = \frac{26}{52} = \frac{1}{2}$.
3. If the probability of rain is 0.7, what is the probability that it will NOT rain? If $P(\text{rain}) = 0.7$, then $P(\text{NOT rain}) = 1 - P(\text{rain}) = 1 - 0.7 = 0.3$.

Solutions for Chapter 21: Exponents and Roots

Solutions

1. Evaluate: $3^4 = 3 \times 3 \times 3 \times 3 = 81$.
2. Evaluate: $10^0 = 1$.
3. Simplify: $x^5 \times x^3 = x^{5+3} = x^8$.
4. Simplify: $\frac{y^7}{y^4} = y^{7-4} = y^3$.
5. Simplify: $(a^4)^3 = a^{4 \times 3} = a^{12}$.
6. Evaluate: $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$.
7. Find the square root: $\sqrt{81} = 9$.
8. Find the cube root: $\sqrt[3]{64} = 4$.
9. Write in scientific notation: $93,000,000 = 9.3 \times 10^7$.
10. Write in scientific notation: $0.000012 = 1.2 \times 10^{-5}$.

Solutions for Chapter 22: Introduction to Functions

Solutions

1. Is the relationship $\{(1, 5), (2, 7), (3, 5)\}$ a function? Why or why not? Yes, it is a function. Each input (1, 2, 3) has exactly one output (5, 7, 5 respectively). It's okay for different inputs to have the same output.
2. Is the relationship $\{(1, 5), (1, 7), (3, 9)\}$ a function? Why or why not? No, it is not a function. The input 1 has two different outputs (5 and 7).

3. For the function $f(x) = 3x - 2$, find $f(5)$ and $f(-1)$.

- $f(5) = 3(5) - 2 = 15 - 2 = 13$.
- $f(-1) = 3(-1) - 2 = -3 - 2 = -5$.

4. For the linear function $y = -\frac{1}{2}x + 3$:

- What is the y-intercept? $b = 3$, so the point is $(0, 3)$.
- What is the slope? $m = -\frac{1}{2}$.

- Create a table of values for $x = -2, 0, 2, 4$.

x	$y = -\frac{1}{2}x + 3$
-2	$-\frac{1}{2}(-2) + 3 = 1 + 3 = 4$
0	$-\frac{1}{2}(0) + 3 = 0 + 3 = 3$
2	$-\frac{1}{2}(2) + 3 = -1 + 3 = 2$
4	$-\frac{1}{2}(4) + 3 = -2 + 3 = 1$

- Describe how you would graph this function using slope and y-intercept. Plot the y-intercept at $(0, 3)$. From there, use the slope: go down 1 unit (because of -1 numerator) and right 2 units (because of 2 denominator) to find another point, $(2, 2)$. Connect these points with a straight line.

Solutions for Chapter 23: Logic and Problem-Solving Strategies

Solutions

1. Is the statement "If a shape is a square, then it has four equal sides" true or false? The statement is **True**. (All squares have four equal sides).
2. Use the problem-solving steps to solve: "You have a bag of marbles. If you double the number of marbles, then subtract 7, you get 21 marbles. How many marbles were in the bag initially?" Let M be the initial number of marbles.
 - **Understand:** Need initial number of marbles. Operations: double, subtract 7, result is 21.
 - **Plan:** Set up an equation and solve it using inverse operations.
 - **Solve:** $2M - 7 = 21$ $2M - 7 + 7 = 21 + 7$ (Add 7 to both sides) $2M = 28$ $\frac{2M}{2} = \frac{28}{2}$ (Divide by 2 on both sides) $M = 14$
 - **Check:** If you start with 14 marbles, double them (28), then subtract 7 (21). This matches the problem.
 - **State:** There were initially 14 marbles in the bag.
3. Describe how you would use the "draw a picture" strategy to solve a problem about finding the shortest path between two points in a maze. You would sketch the maze on paper, representing walls and paths. Then you could draw potential routes, trace dead ends, or use a pencil to mark progress as you try to find the shortest path from the start to the end.
4. A store is having a 20% off sale. You buy a shirt for \$25. The sales tax is 5%. What is the final price you pay for the shirt? (Combine percentages, multiplication, and addition). Original price: \$25. Discount: 20%. Sales tax: 5%.
 - Discount amount: $0.20 \times \$25 = \5 .
 - Price after discount: $\$25 - \$5 = \$20$.
 - Sales tax amount: $0.05 \times \$20 = \1 .
 - Final price paid: $\$20 + \$1 = \$21$.

Solutions for Chapter 24: Financial Literacy Basics

Solutions

1. Your monthly income is \$1,500. Your rent is \$600, food is \$300, transportation is \$150, and entertainment is \$100. How much money do you have left for savings each month? Total expenses = $\$600(\text{rent}) + \$300(\text{food}) + \$150(\text{transportation}) + \$100(\text{entertainment}) = \$1,150$. Money left for savings = $\$1,500(\text{income}) - \$1,150(\text{expenses}) = \350 . You have \$350 left for savings each month.
2. You invest \$5,000 in a certificate of deposit (CD) that offers a simple annual interest rate of 2.5%. How much interest will you earn after 3 years? $P = \$5,000$, $R = 2.5\% = 0.025$, $T = 3$ years. $I = P \times R \times T = \$5,000 \times 0.025 \times 3 = \$125 \times 3 = \$375$. You will earn \$375 in interest.
3. A pair of shoes costs \$80. They are on sale for 25% off. What is the sale price? Discount amount: $0.25 \times \$80 = \20 . Sale price = $\$80 - \$20 = \$60$.
4. If the sales tax is 8%, how much tax would you pay on an item that costs \$50? What is the total cost? Tax amount = $0.08 \times \$50 = \4 . Total cost = $\$50 + \$4 = \$54$.
5. Explain in your own words why having a good credit score is important. A good credit score is important because it shows lenders (like banks) that you are responsible with borrowed money. This can help you get approved for loans (like a car loan or mortgage) at lower interest rates, making them cheaper over time. It can also be a factor in renting an apartment or even getting certain jobs.

Recommended Resources

- **Khan Academy (khanacademy.org):** Excellent free videos and practice problems for all math levels.
- **Cool Math Games (coolmathgames.com):** Fun games to practice math skills.
- **Prodigy Game (prodigygame.com):** Engaging math game for elementary and middle schoolers.
- **Desmos (desmos.com):** Fantastic online graphing calculator for visualizing functions.
- **OpenStax (openstax.org):** Free, open-source textbooks for college-level math.